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TIME, BOHM'S THEORY, AND QUANTUM COSMOLOGY*

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One of the problems of quantum cosmology follows from the fact that the Hamiltonian H of classical general relativity equals zero. Quantizing canonically in the Schrodinger picture, the Schrodinger equation for the wave function Ψ of the universe is therefore the so-called Wheeler-DeWitt equation

$$i\partial\Psi/\partial t = \hat{H}\Psi = 0, \qquad (1)$$

(where $\hat{H}$ is the operator version of H) and we have no quantum dynamics. In particular, it follows directly that, if $\hat{R}$ is the operator representing the radius of the universe,

$$d<\hat{R}>_\psi/dt = 0,$$

where $<\hat{R}>_\psi = \Psi<|\hat{R}|>\Psi$, for *every* Ψ. The universe described by (1) does not expand. But worse still is the fact that there is no time development, contrary to our experience. Like a particle in an eigenstate of zero energy, the state of the universe does not change. The still universe resulting from (1) is a manifestation of the more general "problem of time" afflicting canonical quantum gravity. Despite rare claims to the contrary, most consider it to be a serious difficulty for quantum cosmology.

As we and others have pointed out, Bohm's 1952 interpretation of quantum mechanics offers a neat solution to this problem (see Callender and Weingard 1994, Shtanov 1995, Holland 1993, Vink 1992). Because the "beables" (in Bell's terminology) in Bohm's theory—in this case the components of the metric and other physical fields—obey a different equation of motion than the Schrodinger equation, we can have a nonzero dynamics for the fields, even though the wavefunction does not evolve. In particular, suppose the radius of the universe is the only metric dynamical degree of freedom, i.e., a minisuperspace model, in which $R = \exp[2\alpha]$. Then

$$d\alpha/dt = \partial S/\partial\alpha \qquad (\Psi = R\exp[iS])$$

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can be nonzero even though $d\Psi/dt = 0$. In other words, time is implicitly defined through $\partial S/\partial\alpha$, thereby allowing for a coherent dynamics. The static world of quantum cosmology is allowed to evolve.

However, a natural question to raise about this Bohmian solution (and any other solution to the problem of time) is how the above "Bohm time", $t = t_b$, is related to the time that appears in the Schrodinger equation for the wave function of the quantum matter fields. That is, we know that (in the Schrodinger picture) the wave function of the matter fields obey the Schrodinger equation in terms of what might be called the ordinary "observational" time t. Here t is the time of "classical" quantum mechanics, according to which the metric of spacetime is part of the classical background. Our question is: what is the relation of the observational time t to the Bohm time t_b? Prima facie, there is no necessary connection, which is quite worrying. The purpose of this note is to explore a plausible way these two times can turn out to be the same.

The basic idea of this note is simply to apply Bohm's theory to a significant result of Thomas Banks' (1985). We illustrate the idea using a particle model of Banks that is, in fact, quite similar to the Wheeler-DeWitt equation with α and φ as the dynamical variables. α, recall, is essentially the radius of the universe, and φ is a spatially constant scalar field. Here is what Banks shows. The Hamiltonian has the form

$$H = p^2/2m + mV(\alpha) + p^2/2 + U(\alpha,\varphi) = 0,$$

where on the particle model m is supposed to be the large mass of the particle. Banks then writes the wavefunction as

$$(\alpha,\varphi) = \psi_{WKB}(\alpha)\chi(\varphi,t(\alpha)).$$

To the zero-th order in m, then,

$$(p^2/2m + mV(\alpha))\psi_{WKB}(\alpha) = 0$$

and this implies, to the same order of approximation,

$$i\partial\chi/\partial t_w = (p^2/2 + V(\alpha,\varphi))\chi, \qquad (2)$$

where Banks time $t = t_w$ is defined implicitly through the WKB ansatz as some function of α. It should be emphasized that the full general relativistic treatment follows exactly the same reasoning as the above particle model, so nothing is lost in the simplification.

The idea is that for macroscopic dimensions of $R(\alpha)$, gravity is classical, so the gravitational wavefunction should be of WKB form.[1] But the wav-

[1]In general, there will be two WKB solutions, one positive in the exponent, the other negative. Thus, if $\psi_{WKB}\chi$ and $\psi_{WKB}\chi$ are solutions to the Wheeler-DeWitt equation, so is the superposition $a\psi_{WKB}\chi + b\psi_{WKB}\chi$. Since Banks defines t_w implicitly by the phase of $\psi_{WKB}\chi$, he has to assume that the universal wave function is one of the two components. In the two

efunction χ of the matter field will, in general, not be of the WKB form, for we know the matter fields are not usually in classical form. In the particle model this is reflected in the large mass m of the α particle relative to the φ particle. Although there is no time dependence in the gravitational wavefunction, there is in the matter wavefunction, and so the Banks time, $t = t_w$ *is* the observational time. Therefore (2) is the time dependent Schrodinger equation for χ.

Returning to Bohm's theory, write $\psi = \exp[iS]$, $\psi_{WKB} = R_1\exp[iS_1]$, and $\chi = R_2\exp[iS_2]$, so that

$$\psi = R_1R_2\exp[i(S_1(\alpha) + S_2(\alpha, \varphi))].$$

Then according to the Bohm equation of motion

$$d\varphi/dt_w = \partial S/\partial\varphi = \partial S_2/\partial\varphi.$$

Applying the Bohm equation of motion to (2) directly, we obtain $d\varphi/dt = \partial S_2/\partial\varphi$, since the Bohm time t_b for (2) equals the observational time t by Bohm's original construction. It follows trivially, then, that as far as the field is concerned, the Bohm time t_b and the observational time t coincide.

What about α? Again, from the Bohm equation of motion $md\alpha/dt_b = \partial S/\partial\alpha$, but now $S = S_1 + S_2$ and S_2 depends on α as well as S_1. Therefore

$$md\alpha/dt_b = \partial S_1/\partial\alpha + \partial S_2/\partial\alpha$$

so

$$d\alpha/dt_b = (1/m)(\partial S_1/\partial\alpha + \partial S_2/\partial\alpha).$$

But when $V(\alpha) < 1$, the WKB wavefunction has the form

$$\psi_{WKB} = V(\alpha)^{1/4} \exp[\pm im\int\sqrt{|V(\alpha)|}d\alpha.$$

In that case, $S_1 = \pm m\int\sqrt{|V(\alpha)|}d\alpha$, where the sign is chosen, for example, by boundary conditions (see fn. 1), and

$$d\alpha/dt_b = \sqrt{|V(\alpha)|} + 1/m\ \partial S_2/\partial\alpha.$$

To zero-th order in m, then,

$$d\alpha/dt_b = \sqrt{|V(\alpha)|}.$$

currently popular solutions to the Wheeler-DeWitt equation this is not a problem for us. Hawking's no-boundary wave function is real, so according to Bohm's theory no dynamics is produced for the beables, whereas Vilenkin's boundary condition rules out one of the two WKB components. (See Haliwell 1990 for details about these wave functions.) Note that from a Bohmian point of view, all that is required is that the so-called 'effective' wave function of the universe (see Durr et al. 1992) be of the form $\psi_{WKB}\chi$ and that the total wave function could be a superposition.

Since Banks' implicitly defines the observational time t as

$$d\alpha/dt = \sqrt{|V(\alpha)|} \tag{3}$$

(when $V(\alpha)$ is negative), we have for the time development of α again the result that the Bohm time is the observational time.

Banks' construction demonstrates how the matter fields might enjoy a dynamics even if the wave function of the universe has no time development. For this reason, the reader may wonder why anyone bothers deriving a Bohm time in the first place. Through equations (2) and (3) Banks implicitly defines a time, as we saw, so what do we gain in quantum cosmology by treating it according to Bohm's model? Two remarks are in order. First, it is worth remarking that the Bohmian resolution of the problem of time is much more general. On Banks' model the matter fields only evolve when the wave function is of the WKB form. That is, Banks' time t_w is only defined when the wave function takes this form. But the WKB approximation is only valid when the universe is large and the gravitational fields are nearly classical. By contrast, the Bohm model provides a dynamics so long as the wave function of the universe Ψ is complex. For nearly the same reason, Vink's (1992) elimination of the Bohm time in terms of an internal degree of freedom (a "clock") seems misguided, or at least, unnecessary. Vink finds the Bohm time objectionable because it is essentially the external time of classical mechanics and nonrelativistic quantum mechanics. He feels this is inappropriate for quantum cosmology. Although Vink's procedure for eliminating the Bohm time in terms of a clock is well-defined outside the WKB regime, it still works only when the clock energy is negligible compared to the potential energy. But in view of the Bohm time's greater generality, this attempt seems to mistake a virtue for a vice! Far from being something worth eliminating, the Bohm time not only supplies a coherent dynamics for the universe when the universe is not in the WKB regime but also for when the universe is small compared to Vink's clock.

Second, although the WKB approximation puts the wave function for φ *near* an eigenstate of velocity (differing by an α-dependent factor), it still is not an eigenstate of velocity. This raises the question of its proper physical interpretation. Since Bohm's theory rejects one-half of the eigenvalue-eigenstate link, the approximate velocity eigenstate is physically interpretable on Bohm's theory. More generally, we should note that merely solving the problem of time doesn't entail one has resolved the notorious measurement problem. Left uninterpreted, quantum cosmology still suffers from the problem of making sense of superpositions, a problem which Bohm's theory nicely resolves.[2]

[2]In personal communication with the author we have recently learned that Shtanov (1995) independently derives essentially the same point as this note.

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