XII*—IS TIME 'HANDED' IN A QUANTUM WORLD?

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ABSTRACT In a classical mechanical world, the fundamental laws of nature are reversible. The laws of nature treat the past and future as mirror images of each other. Temporally asymmetric phenomena are ultimately said to arise from initial conditions. But are the laws of nature also reversible in a quantum world? This paper argues that they are not, that time in a quantum world prefers a particular 'hand' or ordering. I argue, first, that the probabilistic algorithm used in the theory picks out a preferred direction of time for almost all interpretations of the theory, and second, that contrary to the received wisdom the Schrödinger evolution is also irreversible. The status of Wigner reversal invariance is then discussed. I conclude that the quantum world is fundamentally irreversible, but manages to appear (thanks to Wigner reversal invariance) reversible at the classical level.

Nature, it is commonly supposed, does not care about right-handedness or left-handedness. According to classical physics, and indeed, according to natural expectations, Nature favours mirror symmetry. That is, Nature does not treat mirror symmetric objects or processes differently. We think this despite overwhelming appearances to the contrary. Most of the objects found in nature are not identical to their mirror image; for instance, my right hand is not identical to its mirror image, my left hand. And most of the right-handed objects and left-handed objects found in nature are not evenly distributed. There are many more right-handed people than left-handed people, many more right-hand twisted seashells than left-hand twisted seashells, and many more left-handed amino acids than right-handed ones. In the face of this phenomena, who do we suspect Nature does not prefer one hand to another?

The short answer is that we believe these asymmetries are accidental. It is a cosmic accident that biological organisms on earth have DNA coiled up in a right-handed way. If we went to other planets and found other biological creatures, we would expect to

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encounter as many planets with left-handed DNA as with right-handed DNA. Likewise for seashells, snails, sugars, etc. Momentarily putting fairly recent experiments in particle physics to one side, we believe these asymmetries are accidental because we believe the fundamental laws of nature are mirror symmetric. If a right-handed sugar can exist, the laws will also allow a left-handed sugar to exist. The initial conditions/laws division marks the distinction between what we deem physically possible and what we deem physically necessary. Claiming the asymmetries are accidental is just our way of saying that they are not derived from law.

Time reversal invariance (TRI) is a temporal analogue of mirror handedness involving a change in the direction of time. TRI is a property of theories such that the laws of nature treat the past and future as mirror images of each other. One way this symmetry is sometimes pictured is by noticing that if we observed a motion picture of classical particles in motion and their interactions, we could not tell whether the film were being run forward or backward. As with mirror symmetry, we tend to believe the world is TRI at bottom despite the mountain of temporally asymmetric phenomena. Milk mixes irreversibly in our coffee; gases spread through their available volumes; the number of McDonald's is constantly increasing. However, at least classically, we judge these asymmetries to be ultimately accidental, just like the twist of snail shells, because we think the fundamental laws are TRI.[1]

Like mirror symmetry, there is some reason to think TRI does not actually hold. Experiments in high-energy physics, coupled with a theorem of quantum field theory, suggest that neutral kaon decay violates this symmetry. However, the standard model in particle physics does not demand lack of TRI—it is merely compatible with it—and the experimental violation of TRI is indirect and very slight. The ultimate cause of this symmetry violation is still very much a matter of debate; crucially, for our purposes, many of the speculations about its origin make it a result of special initial conditions, so it may yet turn out to be a very deep and early cosmic accident. For these reasons, I believe

1. For a stab at how an explanation of the thermodynamic asymmetries along these lines might go, see Callender 1999.

that more work needs to be done before we declare TRI dead due to neutral kaon decay.

In this paper I largely ignore kaon decay and instead consider the possibility that *nonrelativistic* quantum mechanics *already* tells us that Nature cares about time reversal. We will see that in the quantum world the situation described above is inverted, in a sense. In the classical picture we have a fundamentally reversible world that appears irreversible at higher levels, e.g., the thermodynamic level. But in a quantum world we will see, if I am correct, a fundamentally irreversible world that appears reversible at higher levels, e.g., the level of classical mechanics. I consider two related symmetries, TRI and what I call 'Wigner reversal'. Violation of the first is interesting, for not only would it fly in the face of the usual story about temporal symmetry, but it also appears to imply (as I will explain) that time is 'handed', or as some have misleadingly said in the literature, 'anisotropic'.[2] Violation of the second is, as I hope to show, even more interesting. Before investigating the question of whether quantum mechanics implies time is handed, however, we need to discuss two neglected topics: what does it mean to say time is handed and what warrants such an attribution to time?

I

Time-reversal Invariance and 'Handed' Time. To better understand TRI, let us consider mirror symmetry in a bit more detail. Just as there are handed objects in 3-dimensions, there are also 'handed' processes in four dimensions. These are simply the 4-dimensional counterparts of hands. Consider a simple example [See Fig. 1].

Imagine an elementary particle that is (for convenience) shaped like a cube with labels on each of its sides. We subject this cube to an experiment and it responds by emitting a stream of 'X particles' from side A in what we will define as the positive x-direction. The experiment is then repeated, only now with the spatially reflected cube (in the x-direction), which is the mirror image of the original cube. If we likewise reflect all the experimental devices, general symmetry considerations would lead us

2. (An)isotropy is a continuous symmetry corresponding to the operation of a rotation, whereas TRI is a discrete symmetry corresponding to a species of reflection.

Figure 1

to expect X particles to be emitted in the negative x-direction from the reflection of A. Suppose, however, that contrary to our expectations the cube still emitted X particles in the positive x-direction, this time from side B. The X particles behave as if there were something pulling them toward the positive x-direction, which might be, say, the laboratory's right wall. It is wrong to think of the *direction* as the relevant difference here, for we are free to turn this cube around so that the X-particles hit any wall we like. Rather, the relevant difference is that the X-particles are emitted from a *different side* of the cube than we would have expected. Were everything symmetric, the X-particles should leave side A before the reflection and A's image after the reflection.

Although the example is imaginary, it should be remarked that the original 'parity violating' experiments in the 1950's with cobalt nuclei display essentially the same behaviour. In those experiments, electrons are emitted from a radioactive ^{60}Co nucleus and they are 'pulled' to the hemisphere opposite the direction of nuclear spin. That is, the electrons are never emitted in the direction of the nuclear spin, despite this being the parity-transform of the first phenomenon [see Fig. 2].

Returning to the cube example, let us make two assumptions about the experiment. Assume that the two experiments reproduce the mirror processes exactly. No hidden parameters are ignored. Assume also that the difference between these two scenarios is lawlike and that these laws are fundamental ones. The processes described are models of the best scientific theory, whereas the expected mirror-image process has no models in the theory. In short, we are supposing that our best fundamental

Figure 2. State (a) occurs in nature, but (b) does not.

physical theory is representationally complete, so that it is reasonable to believe no 'hidden variables' exist. (Of course, it might also strike one as reasonable to try to design experiments to show that there are hidden factors responsible for the differences; and in reaction to the real parity violating experiments this is exactly what physicists did.)

With these assumptions made, it seems perfectly consistent with scientific practice to hold that the best explanation of these results is that one spatial orientation is preferred. The orientation seems to act as a kind of force, causing the X-particles to leave from one side of the cube rather than another. There are two different physical situations, a cube emitting X-particles to its right from one side and a 'rebuilt' spatially reflected cube emitting X-particles to its right from another side, and *everything else but the spatial reflection remains unchanged.* Moreover, physics assures us that the description is complete and that it is no accident. In such a case we appear justified in claiming that physics in such a world is 'handed', i.e., that one spatial orientation is intrinsically distinguished, via a simple application of Mill's method of difference.[3] In basic structure, this reasoning seems analogous to that leading to the claim that spacetime is curved.

Whether claiming *space is asymmetric* is the right thing to conclude from these hypothetical results is not clear. Is the experiment indicative of space itself being handed, or is it instead evidence of a peculiar property, handedness, had by the cube? Or are these results better construed as evidence of the existence of a 'hand-ordering' field? This question has been a hotly disputed topic ever since Kant argued that the existence of hands implies that space is substantival. Philosophers quickly challenged this inference, but there is still controversy over what to make of handedness. Mill's method apparently warrants positing the existence of some objective property of handedness, but it does not tell us what it is a property *of*. This, I would say, is for physics to decide. If asymmetric space, for instance, is found independently useful to physics, then there may be reason to choose what handedness is a property of. If not, if the handedness hangs useless on the theory like Newtonian gravitational theory's absolute standard of rest, then it would be preferable to try to eliminate it. Or it might be better to resist the very inference to the existence of such a property at all. As Hoefer (1999) argues, there is an explanatory itch here, but it is not clear one gains much by scratching it. Space constraints prevent me from saying more about this inference here, but I can point the reader to Earman 1969 for an expression of a view similar to mine.

It is worth pointing out that in certain cases the inference to the fundamental laws of physics being the culprit (i.e., the source of the asymmetry) rather than asymmetrically distributed initial conditions is practically necessitated. Indeed, this is the case with parity non-conservation. The inference from the radioactive cobalt experiments to the claimed lawlike violation of parity conservation is more complicated and compelling than the above inference in the case of the cube. First, note that parity (non)conservation is not implied by the standard model of particle physics; the standard model is compatible with either result. So how do we know this phenomenon is lawlike rather than factlike? Why think it is due to the laws rather than some asymmetry in

3. More precisely, the experiment's results impose a continuous choice of 'left (right)-handed' versus 'right (left)-handed' orthonormal triads of spacelike vectors at each point.

the matter distribution (say) moments after the Big Bang? The answer does not appeal merely to how 'low-level' the asymmetry is.

Rather, the reasoning contains the following crucial observation (Ballentine 1989). Assuming the asymmetry is factlike means that parity conservation is presumed to hold (that is, $\Pi H = H\Pi$, where H = Hamiltonian, Π = parity operator). But if it holds, it is then quantum mechanically possible for the system to tunnel through the barrier separating the 'right-handed' configuration of radioactive ^{60}Co from the inverted 'left-handed' configuration of radioactive ^{60}Co. Quantum mechanics predicts that the state will tunnel back and forth, cycling at a definite frequency. (In fact, one finds similar cycling between inverted configurations in recent experiments with magnetic ferritin proteins (Awschalom et al. 1995).) And the time it would take for this tunnelling is not very long. The relevance of this fact is as follows. In contrast to ^{60}Co, the potential barrier between a right-twist shell and a left-twist shell is insurmountably high. Tunnelling from one shell to the inverted shell is nomologically possible, but it will take a very long time, even compared with the age of the universe. We are therefore safe in a way we are not with ^{60}Co when assuming that the abundance of right twist shells is due to asymmetric initial conditions. Crudely put, fill a bucket up with right-twist shells, wait a million years, and then test for twist. Quantum mechanics predicts that we will find the same as before. Do the analogue with ^{60}Co, however, and quantum mechanics predicts that if the laws are symmetric, we should get an approximately equal number of 'right' states and 'left' states. Since we do not get this, we are not free to suppose the laws concerning ^{60}Co are symmetric—at least not without making a mockery of the quantum mechanical laws dictating tunnelling times.

Turning to time, the concepts of temporal anisotropy or handedness and TRI are simply the temporal counterparts of spatial handedness and parity invariance, respectively. The spatial case is the process with the spatial order inverted; the temporal case is the story told with the temporal order inverted. Relative to a co-ordinisation of spacetime, the time reversal operator takes the objects in spacetime and moves them so that if their old co-ordinates were t, their new ones are –t, assuming the axis of reflection is the co-ordinate origin. This operation is a discrete improper transformation.

With this understanding of the time reversal operator, we can now formulate a definition of TRI:

> A theory is TRI just in case given a lawful sequence of states of a system from an initial state S_i to final state S_f with chance equal to r, the sequence from the temporally reflected final state S_f^T to the temporally reflected initial state S_i^T, also has chance equal to r, i.e., $P(S_i \rightarrow S_f) = P(S_f^T \rightarrow S_i^T)$.

Superscript '^{T}' denotes the action of the time reversal operator that temporally reflects the state. As I understand it here, '^{T}' switches the temporal order by switching the sign of t. It also switches the sign of anything logically supervenient upon switching the sign of t, e.g., the velocity $d\mathbf{x}/dt$. But that is all '^{T}' does.[4] And I have included reference to the probability the theory says the process has of going from one state to another to take care of indeterministic theories.

Other definitions for probabilistic TRI have been proposed, but I believe that they are all inadequate. For instance, it might be said that an indeterministic process is TRI just in case if $S_i \rightarrow S_f$ is compatible with the laws then so is $S_i^T \rightarrow S_f^T$. This is the probabilistic version of what is sometimes called 'motion reversal invariance'. Motion reversal invariance holds that if state S_i evolves to state S_f, then it is also possible for the state S_f^T to evolve to S_i^T. However, it is too easy for a probabilistic theory to be TRI according to this conception, for rarely will stochastic theories specifically disallow the temporally inverted process. Probabilistic theories may not constrain the evolution of a system to a unique trajectory, as in deterministic theories, but that hardly means that 'anything goes' during probabilistic evolution. What we want from TRI theories, after all, is that they 'say the same thing' in both directions of time. If the theory gives chances of a forward transition occurring, it ought to give the same chances of the inverted one if it is TRI.

As in the spatial case, we will say 'time is handed' if the fundamental laws of nature are not TRI. 'Handed' is perhaps not the

4. David Albert, forthcoming, argues—rightly in my opinion—that the traditional definition of TRI, which I have just given, is in fact gibberish. It just does not make sense to *time-reverse* a truly *instantaneous* state of a system. Nevertheless, for the sake of brevity I stick with the usual definition; by curtailing the effect of ^{T} so that it only effects those changes logically necessitated by $t \rightarrow -t$ we limit its harm.

best term, but it is not as inaccurate as the traditional 'anisotropic' and not as cumbersome as 'preferential'. By 'handed time' I mean that one direction of time is preferred in the sense that one hand may be preferred over the other. The fundamental laws of motion in Newtonian mechanics are TRI and so time is not handed in a classical world (Callender 1995).

II

Penrose's Thought Experiment. I want to begin by making one of those points that is obvious once you hear it but none the less necessary to make since it is so commonly ignored. I will do so in the context of a well-known thought experiment by Penrose 1989. Penrose and others suggest that this experiment displays the time-asymmetry of quantum mechanics.

Consider the following idealised experiment, depicted in Fig. 3. At L we place a source of photons—a lamp—that we direct precisely at a photon detector—a photocell—located at P. Midway between L and P is a half-silvered mirror tilted at 45 degrees

Figure 3

from the line between L and P. Speaking loosely, when a photon's wavefunction hits the mirror it will split into two components, one continuing to P and the other to a perpendicular point A on the laboratory wall. Since the wave function determines the quantum probabilities, and by assumption it weights both possibilities equally, we should expect one-half of the photons aimed from L to make it to P and one-half to be reflected to A. Each photon has a one-half chance of either being reflected to A or passing through to P.

Penrose's argument consists of a comparison of two transition probabilities: 'Given that L registers, what is the probability that P registers?', i.e., P(P, L), and 'Given that P registers, what is the probability that L registers?', i.e., P(L, P) (358). As we know, the value of the first one, P(P, L), is one-half: photons released from the lamp have a one-half chance of reaching the photocell. However, to the second probability Penrose assigns the value of unity. Because he considers these questions the time reverses of each other, he takes this as evidence of the time asymmetry of quantum mechanics.

Why does Penrose assign unity to the second probability? He explains, 'If the photocell indeed registers, then it is virtually certain that the photon came from the *lamp* and not from the laboratory wall! In the case of our time-reversed question, the quantum-mechanical calculation has given us *completely the wrong answer*!' (358). His idea seems to be that if P registers, it must have come from L, because L is a lamp and A is a wall. Our knowledge of where the photon must have originated allows us to draw on information superior to that provided by quantum mechanics, thereby enabling us to give the second probability a different value.

Penrose's argument appears confused. Why use extra-quantum mechanical information when judging whether the theory is time reversible? More importantly, why compare $P(S_i \rightarrow S_f)$ with $P(S_f \rightarrow S_i)$ *and not with* $P(S_f^T \rightarrow S_i^T)$? If Penrose is genuinely concerned with TRI, he should treat the emitter as a receiver and vice versa. The time-reverse of the question of the probability of P, given L, is not the probability of L given P. Rather, it is 'what is the probability of the time-reverse of L given the time-reverse of P?'. That is, what is the probability that the time-reversed photocell will emit a photon to the time-reversed lamp?

While these questions are important to ask of Penrose's argument, the point I wish to make is the following. As Savitt 1995 shows, one can easily build a classical version of the experiment, with exactly the same frequencies as in Penrose's experiment, even though classical mechanics is the paradigm of time symmetry. Just replace the mirror with a box containing a shutter that spends half of its time at one position and half at another with the two outputs varying with the shutter position. This observation leads to the obvious but neglected one: *the observed frequencies by themselves tell one nothing about whether the theory is TRI or whether time is handed.* For the former, we need to know if the temporally asymmetric frequencies are a matter of law, and for the latter, we need to know if they are fundamental laws. One can find all sorts of 'asymmetric phenomena' in quantum mechanics, but so too can one find this phenomena in classical physics. To see if quantum mechanics is interestingly different than classical mechanics in this regard, we therefore need to look at the laws and ontology of the theory.

III

Quantum Mechanics. Bearing this point in mind, we are now ready to see whether quantum mechanics is TRI. Quantum mechanics has two sorts of predictive algorithms that will concern us. The first is the Schrödinger equation

$$i\hbar \frac{\partial \psi}{\partial t} = H\psi \qquad (1)$$

where ψ is the wavefunction of the system and H is the Hamiltonian. The Schrödinger evolution is perfectly deterministic. This equation is a fundamental law of nature according to all interpretations of non-relativistic quantum mechanics. The second algorithm is a probabilistic one, one that assigns the chances of finding particular values of an observable upon measurement. The state ψ can be decomposed in any orthonormal basis

$$\psi = \sum_i c_i \psi_i$$

where the ψ_i, the eigenvectors of the relevant observable, represent all the possible values of some experimental question (and for simplicity only, we have restricted ourselves to observables with a discrete spectrum). The modulus squared $|c_i|^2$ of the coefficient of each eigenvector gives the probability of finding that particular value. For ideal measurements the logical form of this algorithm is

(FOR) P(observe at t_2 value a_i of observable A, ψ at t_1) = p.

where $t_2 > t_1$. Thus, quantum mechanics gives us conditional probabilities in what Sober 1994 has called 'temporally oriented laws', i.e., laws that give the probability of some later state occurring given that some earlier state occurred.

IV

Is Quantum Mechanics Symmetric Under TRI? Part I. Let us consider the probabilistic algorithm first. Here we can see that Penrose, despite the odd way he puts the point, may none the less have been on the right track, for quantum mechanics gives us forward transition probabilities like (FOR) but not backward transition probabilities like

(BAC) P(observe at t_0 value a_i of observable A, ψ at t_1) = p′.

Quantum mechanics does not give us the probability of earlier states, given later states. The theory is predictive but not retrodictive. And if one tried to apply BAC to quantum measurements it would typically give the wrong answer.

Furthermore, it is a well-known fact (to my knowledge first discussed by Watanabe 1965) that one in general *cannot add* non-trivial backward transition probabilities to a theory with non-trivial future transition probabilities (without making the theory useless). Specifically, if one adds BAC to a theory with FOR (or vice versa), assumes BAC and FOR are stable with respect to time (time translation invariant), and gives them non-trivial values, then one is forced by Bayes theorem to assign to each state an *unconditional probability* of being in that state.[5] This

5. Bayes theorem allows one to transform BAC probabilities into FOR probabilities multiplied by prior unconditional probabilities. Using this fact and the assumption that BAC and FOR probabilities are time translation invariant gives us our result. Here is a proof for a system that can occupy two states, A and B, where $t_3 > t_2 > t_1$ (see Sober 1994). The proof can easily be generalized to any finite number of states and times. Suppose we are in state A at t_2 and we add BAC transition probabilities, $P(Bt_1/At_2)$ and $P(At_1/Bt_2)$. By time translation invariance, we know $P(Bt_1/At_2)/P(At_1/At_2) = P(Bt_2/At_3)/P(At_2/At_3)$. By Bayes theorem this gives $P(At_2/Bt_1)P(Bt_1)/P(At_2/At_1)P(At_1) = P(At_3/Bt_2)P(Bt_2)/P(At_3/At_2)P(At_2)$. But FOR and

means the expected state of a system is the same for all time. This may suffice for describing certain types of motion, e.g., Brownian motion, but it will not suffice for any more interesting physics. In addition, the theory would be odd in another more philosophical respect, as Healey 1981 points out. If FOR and BAC are considered lawlike, then so are the unconditional probabilities we derive (since they follow directly from the laws and the probability calculus). So a theory that incorporated BAC with FOR would dictate, as a matter of law, the unconditional probability of every state—in effect, it would make the boundary conditions nomological.

One way of avoiding this result is as follows. Instead of considering two times, t_1 and t_2, and adding BAC and FOR between these times, take three times and add BAC and FOR as the probabilities for transiting to a *third* state t *between* t_1 and t_2. By considering three times and the probability for intermediate states Aharonov, Bergmann and Lebovitz 1964 (ABL) were able to discover the so-called ‘time symmetric’ formalism of quantum mechanics. In this formalism one is given the probability of some outcome at t conditioned on the earlier state at t_1 *and* the later state at t_2. This formalism has been useful for seeing a number of experimental results, but—despite appearances—I do not believe it is so useful for our discussion. Put briefly, the ABL formalism is just a novel reformulation of the mathematics of quantum mechanics. It is not a new interpretation or formal extension of quantum theory. As such, it cannot tell us whether BAC or FOR is a fundamental law. Furthermore, the interpretation of the probabilities arising from this formalism for intermediate states is riddled with controversy. Until the formalism is properly interpreted, efforts to make it answer our question will be in vain.

The relationship between the time symmetric formalism and the usual ‘predictive’ one is interesting, however. If one introduces ‘coherence-destroying’ measurements after the measurement at t, thereby making the measurement at t independent of the one at t_2, the time symmetric formalism reduces to the usual one with FOR and not BAC. And if one introduces ‘coherence-destroying’ measurements before the measurement at t, thereby

time translation invariance gives us $P(At_2/Bt_1)/P(At_2/At_1) = P(At_3/Bt_2)/P(At_3/At_2)$. Comparing equations, we find that the expected state does not change with time: $P(Bt_1)/P(At_1) = P(Bt_2)/P(At_2)$.

making the measurement at t independent of the one at t_1, the time symmetric formalism reduces to a 'retrodictive' one using BAC and not FOR. But as it happens, in our world, we do not need the time symmetric formalism, nor can we use the retrodictive one except for in artificial cases. Our world is effectively randomised after measurements rather than before. ABL believe this provides a 'thermodynamic' explanation of the origin of the FOR and not BAC asymmetry. Certainly it is suggestive of such an explanation. But until one solves the measurement problem, wherein one sees clearly the status of FOR, this point will remain merely suggestive.

Returning to the main thread, we can see that if BAC cannot be added to FOR in quantum mechanics (as I believe to be the case), then the theory is not TRI if FOR is a law of the theory. The reason is simply that quantum mechanics does not say the same thing in one direction of time as it does in the other. Indeed, in this case it says one thing in one direction of time but *nothing* in the other. More precisely, the $P(S_i \rightarrow S_f) \neq P(S_f^T \rightarrow S_i^T)$ since the latter probability is not even defined (and cannot be added).

By itself, however, the lack of TRI of this algorithm does not prove anything about whether time is handed or not. Remembering our earlier warning, we know we must determine whether or not this algorithm is a fundamental law of nature or not. Only if it is are we justified in asserting that time is handed.

It is at this point that the interpretation of quantum mechanics shows its face. (The reader can consult Albert 1992 for an introduction to these interpretations.) According to some interpretations, such as Bohm's, FOR is mererly derived from the laws and initial conditions; according to others, such as GRW, FOR is a fundamental law. In Bohmian mechanics there is a dual ontology with two fundamental laws. Wavefunctions exist and are governed by the Schrödinger equation, and always-determinate particles exist and their motion is governed by the so-called guidance equation $\mathbf{v} = \mathrm{Im}\nabla\psi/\psi$. The guidance equation is TRI invariant if the Schrödinger equation is. Assuming for the moment the conventional wisdom that the Schrödinger evolution is time symmetric, this implies that the fundamental laws in a Bohmian universe are TRI. The observed asymmetry is due to asymmetric initial conditions (see Arntzenius 1997 for more). There would be no reason to explain the FOR-BAC asymmetry in terms of time's handedness. In a sense, Bohmian mechanics provides precisely the thermodynamic explanation of the asymmetry suggested by the ABL formalism.

By contrast, there is reason to blame time's handedness according to many other theories. For GRW theory, and other collapse theories, the FOR-BAC asymmetry is a direct result of the non-TRI of the collapse mechanism. On collapse theories, a certain feature of the system (e.g., particle number, mass, being observed) will trigger a non-unitary, indeterministic transition from ψ to one of its components ψ_i. This collapse is not governed by the Schrödinger evolution. And in general there is no way of evolving from the collapsed system back to the uncollapsed system with the same chance. According to collapse theories, there is a preferred orientation to time.

Less expected, perhaps, is that no-collapse theories such as the many-minds and modal interpretations also pick out a preferred direction to time. According to many minds (at least the version I understand), minds are not quantum mechanical entities (they do not superpose). They are like Bohmian particles; but unlike Bohmian mechanics, many minds does not explain the FOR-BAC asymmetry with asymmetric initial conditions. Rather, it explains the asymmetry with a new law of transition for the minds, one that states something very much like FOR, specifically, that the probability of one of one's minds being in state ψ_i is given by $|c_i|^2$. The modal interpretation does exactly the same for its so-called 'value states'.

We have found that according to all the major interpretations except for Bohmian mechanics, time is handed. But notice that this handedness is very theoretical in nature. This is clear from comparing many minds theory with Bohmian mechanics. Which of these two theories is true (if one is) is completely underdetermined by experiment. In principle, no experiment can (dis)-confirm one without doing the same for the other. Yet the two differ as to whether time is handed or not. One explains the FOR-BAC asymmetry with asymmetric laws, the other with asymmetric initial conditions, yet there is no possible experiment that could tell us which is the right explanation. Collapse theories, by contrast, are in principle distinguishable from Bohmian mechanics, so in this sense we could experimentally verify time's handedness by establishing the truth of collapse theories over Bohm's theory. Let us now turn to the Schrödinger evolution...

V

Is Quantum Mechanics Symmetric Under TRI? Part II. The Schrödinger equation (1) is a first-order partial differential equation where H is a real-valued quantum Hamiltonian. Time reversal brings out a minus sign

$$-i\hbar\frac{\partial\psi}{\partial t} = H\psi. \tag{2}$$

(2) is clearly not equivalent to (1), despite frequent claims to the contrary. The equation is therefore non-TRI. Gaussian wavepackets, for instance, will have their width spread with increasing time. Their time reverses will evolve according to (2) and not (1)—so they will not evolve the same way in both temporal directions—and this will prevent them from evolving to the time reverse of the initial wavepacket. The theory is non-TRI in just the way classical mechanics would be if $F = m\mathbf{v}$ were the fundamental law rather than $F = m\mathbf{a}$.[6]

The standard response to this observation, due to Wigner 1936, is that in quantum mechanics time-reversal is implemented not merely by inverting the order of time, but also by operating on the state with an anti-linear operator. In the co-ordinate representation this operator is identical to '*' which takes the complex conjugate of the state. If we invert the temporal order *and* operate on the state with *, then we get back

$$i\hbar\frac{\partial\psi^*}{\partial t} = H\psi^*$$

which is of the same form as the original Schrödinger equation (1). Thus (1) enjoys the following symmetry: $\psi(x, t)$ is a solution iff $\psi^*(x, -t)$ is. Let us dub this symmetry 'Wigner TRI' or WRI. The Schrödinger equation is WRI but not TRI.

If one surveys the literature concerning this issue, one finds many arguments that attempt to blur the difference between WRI and TRI. Probably the most frequent claim is that in quantum mechanics the physical content is exhausted by the probabilities. As Davies puts it, 'a solution of the Schrödinger

6. The non-TRI of the Schrödinger equation is further discussed (independently) in Callender 1997 and Albert, forthcoming.

equation is not itself observable' so Wigner's operation can restore TRI while leaving 'the physical content of QM unchanged' (Davis 1964, 156). The idea is simply that the observed configurations can only tell us about the absolute value of ψ, and not about ψ itself. That is, since the predictions are made using Born's rule, $\rho = |\psi|^2$, the probability that a state ψ will have a certain value equals the probability that a state ψ^* will have that same value. The response to this claim is that, although it is true that the observable content of the theory is given by Born's rule, unless we resort to operationalism or verificationism, this is not relevant. Arguably, according to all the major interpretations of quantum mechanics, ψ is a genuine part of the ontology of the quantum world.

This is not the place to go through all the misguided attempts to blur the distinction between WRI and TRI, but another popular argument claims that WRI is necessitated by the need to switch sign of momentum and spin under time reversal.[7] Here the reply is that there is no such necessitation. In quantum mechanics, momentum is a spatial derivative ($-ih\nabla_x$) and spin is a kind of 'space quantization'. It does not logically follow, as it does in classical mechanics, that the momentum or spin must change signs when $t \rightarrow -t$. Nor does it logically follow from $t \rightarrow -t$ that one must change $\psi \rightarrow \psi^*$.

It should be clear that the two symmetries are indeed different simply because a symmetry is defined by its operations on states. Because the two operations are different, the two symmetries are different. They are also different with respect to time's handedness. Our argument in Section II from fundamental non-TRI laws to time's handedness only worked because the sole possible culprit for the different physical outcomes in the temporally reflected case is the temporal reflection. The failure of WRI, however, would implicate *either* temporal reflection or complex conjugation. Since the latter does not involve time, we cannot infer from the lack of WRI that time has a preference.

However, we can infer that time is handed from the non-TRI of the Schrödinger equation, for all interpretations take this law

7. For detailed treatments of TRI in quantum mechanics, see Roman 1950 and Sachs 1987. Roman, I think, approaches the issue from the right direction, but even he admits 'we should like, *even if only by some artifice*, to achieve full covariance [TRI]' (266, emphasis mine). For discussion see Callender 1997.

as fundamental. That the equation is invariant under the operation of some other symmetry transformation does not affect our inference. Therefore, if this reasoning is correct, time is handed according to all interpretations of QM, even Bohmian mechanics.

VI

Is Quantum Mechanics Symmetric Under WRI? The Schrödinger equation is WRI, but that does not imply quantum mechanics is. Some interpretations of the theory modify or interrupt the Schrödinger evolution, others do not. Before directly considering this issue, however, I would first like to argue that quantum mechanics *must* be at least *approximately WRI*. We will answer our question by claiming that most interpretations of the theory satisfy WRI, and for those that do not, they are approximately WRI. By giving the general reason why all interpretations must be approximately WRI, we will shed light on questions such as why in quantum mechanics we reverse the sign of the momentum and the spin during time reversal, even though neither of these variables logically depends on the time co-ordinate. In other words, the argument that follows can be used as a way of understanding why WRI, though not equivalent to TRI, is nevertheless important—arguably more important than TRI.

The argument is simple. Quantum theory, on any interpretation, must at the classical limit correspond to classical mechanics. At the appropriate scale quantum mechanics must approximately reproduce the phenomena that appear to obey classical mechanics. But classical mechanics is TRI. If trajectory $\mathbf{x}(t)$ is permitted by the laws then so is the temporally inverted trajectory $\mathbf{x}(-t)$. We can observe this reversibility, at least in certain cases. The thermodynamic asymmetry limits our ability to see this, but it does not eliminate it. We can easily verify that (say) a billiard ball going from left to right around an attractive central force can be made to go from right to left if the forces and initial position of the ball are reversed. Thus quantum theory must approximately reproduce these phenomena at the classical scale.

An apparently necessary condition for quantum mechanics to reproduce these phenomena is that it be symmetric under change of sign of its momentum when one goes from $t \rightarrow -t$. Let us see why.

The classical correspondence of quantum mechanics is, as the cognoscenti know, a very delicate and murky issue. In standard quantum mechanics, there is not (to my knowledge) any truly satisfying approach to this correspondence. Nevertheless, there is a very simple relationship between classical mechanics and quantum probabilities that will allow me to make my point, namely, Ehrenfest's theorem. Despite its severe limitations in understanding the classical limit, it does have the virtue of being true. Roughly speaking, this theorem states that the centroid of the quantum mechanical probability distribution will follow a classical trajectory. That is, it says the classical equations hold for average quantum values. Thus for quantum operators **x** and **p** we have

$$\begin{aligned} \frac{d\langle x\rangle}{dt} &= \frac{\langle p\rangle}{m} \\ \frac{d\langle p\rangle}{dt} &= -\langle \nabla V(x)\rangle = \langle F(x)\rangle \end{aligned} \tag{3}$$

which, when combined, yield an analogue of Newton's law of motion

$$m\frac{d^2\langle x\rangle}{dt^2} = \langle F(x)\rangle$$

To really play a role in the classical correspondence, we would need to show that $F(\langle \mathbf{x}\rangle)$ approximates $\langle F(\mathbf{x})\rangle$. For many reasons we often cannot do this, and worse, sometimes we can and still end up with a distinctly quantum mechanical system, e.g., the quantum oscillator. In any case, Ehrenfest's theorem does establish a link between classical mechanics and quantum probabilities, and this is all I need. I would expect the following sort of argument to hold in superior accounts of the classical correspondence, too.

Concentrate on equation (3). Inverting the order of time slices—that is, performing a time reversal—brings out a minus sign on the LHS

$$-\frac{d\langle x\rangle}{dt} = \frac{\langle p\rangle}{m}$$

Unless the RHS of this equation also brings out a minus sign the reverse averaged trajectory $\langle \mathbf{x}(-t)\rangle$ will not be lawful. But it had better be—at least approximately—since we know both **x**(t) and **x**(–t) are classically possible and we would expect the quantum versions of these trajectories—at least on average—would also be lawful. But the RHS does not have the right kind of time-dependence to do this, since

$$\langle p\rangle = \int \psi^*(x,t)\left(\frac{\hbar}{i}\right)\frac{\partial\psi(x,t)}{\partial x}\,dx$$

So our equation is not invariant under time reversal. For the sake of the classical correspondence, however, we *need* it to be invariant under *some* symmetry. It is obvious that the equation is invariant under the combined transformation of $t \to -t$ *and* $ih\,\partial/\partial\mathbf{x} \to -ih\,\partial/\partial\mathbf{x}$. Switching the sign of the quantum momentum, therefore, is necessitated by the need for quantum mechanics to correspond to classical mechanics.

From this conclusion we can also see that to mirror the classical time-reversed trajectory the Schrödinger equation must be invariant under WRI. Consider the commutator

$$[x, p] = i\hbar$$

and the operator, call it T, implementing the required transformations

$$TxT^{-1} = x$$

$$TpT^{-1} = -p$$

Letting T act on the commutator yields

$$x(-p) + px = TiT^{-1}\hbar$$

But this reduces to the original commutator iff $TiT^{-1} = -i$. In other words, the commutator is T-invariant only if T is anti-linear. And this means T must involve complex conjugation, i.e., WRI. (The exact form of T varies with basis.) We have already seen that the Schrödinger equation is invariant under T: $\psi(\mathbf{x}, t)$ is a solution iff $\psi^*(\mathbf{x}, -t)$ is. From this it follows that the Schrödinger evolution is WRI. As far as the trajectories in x-space for classical objects are concerned, trajectories are fully reversible.

Let us briefly return to the interpretation of quantum mechanics. We looked at three different types of interpretation of the theory: 1) hidden variable interpretations, 2) collapse interpretations, and 3) Everett-style interpretations. Since the Schrödinger evolution is WRI, interpretations of type 3 will be WRI if they add nothing to the Schrödinger evolution. The first type adds something to the Schrödinger evolution. However, these additions are usually tied to the quantum state in some way and enjoy the same symmetries as the quantum state. For instance, in Bohmian mechanics the velocity of the particles is a function of the quantum state, and since the state is insensitive to complex conjugation so too is the velocity.

The second type of interpretation, however, is typically not WRI. The Schrödinger evolution is interrupted according to these interpretations. When the wavefunction collapses to one of the measurement observable's eigenstates, there is in general no way back to the original uncollapsed state. This leads to the question of how this squares with the observed TRI of classical mechanics.

To briefly illustrate in the space I have left, consider the question in a worked out theory of collapse, GRW. In the original version, the wavefunction is 'hit' at certain intervals by Gaussian functions that shrink the wavefunction to negligible amplitude in all but one eigenstate (plus small tails). The number of particles determines the likely frequency of the 'hits' in the system: the more particles, the more hits. And the likely hit location on the wavefunction is dependent upon the size of the amplitude: the wavefunction is most likely to be hit by a Gaussian centred on locations where the amplitude is greatest. Now consider a classical system, say, a basketball. Its wavefunction will be very well localised. According to GRW it will be hit approximately every 10^{-8} s or more with very high likelihood at the centre of the wavepacket. Recalling Ehrenfest's theorem, we thus know that the GRW basketball will evolve approximately classically. The theory is approximately WRI. It is only not WRI sometimes at the microlevel. And it is hoped that these non-WRI collapses will be observed (or not) in crucial tests in the near future. Non-WRI, unlike the possible non-TRI we discovered in Section V, is in principle directly observable.

VII

Conclusion. Time in a quantum world is handed. In many interpretations it is handed due to the noninvariance of its probabilistic algorithm under time reversal. However, some interpretations, like Bohm's, evade this conclusion by not viewing this algorithm as a fundamental law of nature. But all interpretations of the non-relativistic theory take the Schrödinger equation to be fundamental. This evolution is not TRI, contrary to received wisdom, so time in a (nonrelativistic) quantum world is handed. Exploring further what this means must be left for another time. But we can note our arrival at a curious inversion of the situation described in the introduction. We once thought that all irreversibility was contingent, due only to initial conditions, and that fundamentally the world was reversible. Nature hid her true reversibility behind the temporally asymmetric processes familiar from experience. Now with quantum mechanics, we find Nature being doubly deceptive: our fundamentally irreversible world disguises itself as reversible one that in turn masks itself as irreversible.[8]

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