No Time for Time from No-Time

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Abstract

Programs in quantum gravity often involve formalisms that are supposedly fundamentally timeless, with physicists claiming that time emerges from fundamentally timeless physics. In the semiclassical time program time arises only after approximations are taken. We show that the usual physical justifications for these approximations assume that time already exists. The semiclassical time approach turns out to be either unjustified or circular in deriving time from no–time.

1 Introduction

Programs in quantum gravity often result in formalisms that are said to be fundamentally timeless. Because we observe change, it's important that such programs recover time from no-time in some way. One popular idea is that time emerges from fundamentally timeless physics, like how macroscopic color arises from fundamentally uncolored basic physics. An example is the semiclassical time program in canonical quantum gravity, where the key idea is that time emerges from fundamentally timeless physics as a result of semiclassical approximations. Nothing at the fundamental level supposedly plays the time role throughout any solution, but time emerges in some sectors in the state space of some solutions which are approximately classical.

However, the comparison with color suggests an obvious worry: circularity. Physically, color only emerges from uncolored matter diachronically. Color arises from the interaction of observers like us with matter behaving a certain way across a temporal interval. Replace color with time and the threat is obvious: if time emerges from no-time but the emergence itself requires time, then we can't really say we've derived time from no-time. Time emerges if we blur our vision, but if blurring takes time then time never disappeared.

We want to raise this objection in a very sharp way for the semiclassical time program by focusing specifically on the approximations necessary to derive time from no-time. We argue that time implicitly sneaks its way back in via the physical justifications behind these approximations. If one is allowed to assume anything, one can derive any equation from any other. Physical assumptions need physical warrant. If, as we will argue, that warrant assumes time exists, then either we were unjustified in applying the approximations because we were applying them to timeless solutions, or the derivation succeeds on pains of circularity. We believe other programs claiming to derive time from no-time may also be susceptible to the same worry, but we will not demonstrate that here.

2 The Problem of Time and Emergence of Semiclassical Time

Quantum gravity seeks to reconcile our best theory of gravity, general relativity, with our best theory of matter, quantum theory. Many different strategies exist for doing so, e.g., string theory, but here we focus on the oldest canonical approach, quantum geometrodynamics, and its recovery of semiclassical time. We do so because this recovery has been rigorously developed. We expect, however, that many of our lessons will generalise elsewhere.

Canonical approaches employ a Hamiltonian formalism that is then quantized. To do this for gravity, one must cast general relativity into its Hamiltonian "3+1" form, where one decomposes spacetime into leaves of spacelike hypersurfaces. The Hamiltonian framework demands canonical variables and conjugate momenta. For gravity, the basic variable is the three-dimensional spatial metric characterizing the spacelike hypersurfaces and the conjugate momentum is defined in terms of the trace of the spatial metric's extrinsic curvature. In classical particle mechanics the Hamiltonian governs the spatial configuration of particles through time; in classical Hamiltonian general relativity, the Hamiltonian governs the spatial geometry itself through time. Once cast in this form, it is then time to quantize.

The counterpart of the quantum state is a functional operating in a configuration space of spatial three-metrics. To quantize, we turn the variables into operators. Trouble arises, however, because general relativity is a constrained Hamiltonian system. One of the constraints arises due to the time reparameterization freedom we have in general relativity – we can slice up or foliate spacetime in many different ways. This constraint, the Hamiltonian constraint, demands that the Hamiltonian vanish. If we follow Dirac's procedure when quantizing a constrained system, the constraint is imposed as a restriction on physically possible wave-functions. Intuitively, the idea is that the operator shouldn't rotate the quantum state into physically unrealizable states. In geometrodynamics, making the Hamiltonian an operator and imposing the constraint yields:

$$\hat{H}\Psi(h_{ab}(x), \phi) = 0 \tag{1}$$

i.e., the famous Wheeler-DeWitt (**WD**) equation. $\hat{H}$ is the Hamiltonian operator for both gravity and matter, and Ψ is the **WD** wave-functional which depends on the spatial threegeometries encoded by the spatial metric $h_{ab}(x)$ and whatever matter fields we include, e.g., ϕ (for simplicity, ϕ is usually a massive scalar field).

The core idea of the semiclassical time program is that time emerges if the gravitational wave-functional is semiclassical. If it is not — if the gravitational field is quantum — then the concept of time will not find any realizer. This idea was expressed by DeWitt (1967) but developed by Banks (1985) in the canonical approach.[1] The **WD** wave-functional is, at the fundamental level, utterly timeless. But it nonetheless describes patterns of correlations, just like a checkered shirt at an instant contains a spatial pattern of correlations amongst stripes and colors. In the semiclassical interpretation, the idea is that at a certain level of approximation, a pattern of correlations "looks" temporal, just as a checkered shirt can look solidly colored if one zooms out far enough.

By "looks temporal" we mean that a parameter plays the time role. While defining the time role could become quite messy and philosophical, this program adopts a very minimal sufficient condition that seems quite plausible, namely, that something plays the time role if it behaves as the "t" does in the ordinary time-dependent Schrödinger equation (**TDSE**). In other words, if the matter fields depend on or vary with some parameter in the same way as they depend on or vary with the "t" in the **TDSE**, that warrants calling that parameter time.

Herein lies the key achievement of the semiclassical time program: given suitable approximations, they show that the non-temporal gravitational fields h_{ab} can play the role of time in a functional Schrödinger equation for the matter fields ϕ. If one approximates from the **WD** equation appropriately, it looks like matter is evolving with respect to time (*a la* a Schrödinger equation) against a classical gravitational curved spacetime background (described by the semiclassical Einstein-Hamilton-Jacobi equation).

Let us now turn to the actual derivation of time and the functional Schrödinger equation. Here we loosely follow a presentation by Derakhshani (2018) (but see also Banks (1985, Appendix B)). The derivation has two crucial steps. One, it uses a Born-Oppenheimer ansatz and writes the wave-functional of the universe in product form. Two, it uses a **WKB** approximation on the gravity term in this product. The reader can think of the first move as separating out a sub-system from the total system. The second move then shows that when that sub-system behaves approximately classically, it can function as a clock for the rest of the system.

We begin with a wave-functional that satisfies the **WD** equation

$$\Psi(h_{ab}, \phi) \tag{2}$$

and other necessary constraints. This describes a static wave in a high-dimensional configuration space.

[1] See e.g. Kiefer 2004, Anderson 2007.

Since h_{ab} depends on the Planck mass m_p, which is extremely small, the idea of separating scales via a Born-Oppenheimer (**BO**) approximation is natural. This is because quantum gravitational effects aren't expected except near the Planck scale and h_{ab} depends on m_p. Hence we can separate the "heavy" part of the wave-function, $\chi(h_{ab})$ from the "light" part, $\psi(\phi, h_{ab})$:

$$\Psi \approx \chi(h_{ab})\psi(\phi, h_{ab}) \tag{3}$$

The idea will be to use the h_{ab} degrees of freedom as a clock for the light part ϕ.

In a **WKB** approximation, we first substitute the ansatz Ae^{iS} for a wave-function. We do that for the first factor, the heavy subsystem, turning the wavefunction into

$$\Psi \approx A(h_{ab})e^{im_p^2 S(h)}\psi(\phi, h_{ab}) \tag{4}$$

Next, expand S(h) as a power series in m_p^2:

$$S = m_p^2 S_0 + S_1 + m_p^{-2} S_2... \tag{5}$$

Then, as usual in **WKB**, we plug the S_0 and S_1 terms each back into the wave equation and solve. In the ordinary quantum mechanics case, the 0^{th} order terms returns a Hamilton-Jacobi equation and the 1^{st} order term returns a continuity equation. Essentially the same happens here. In particular, solving to leading order m_p^2, we derive a semiclassical gravitational Hamilton-Jacobi equation.

Take a solution S of this set of equations. Based on experience with geometric optics and quantum theory, we know S defines in superspace a vector field ∇S whose integral curves can be parametrized by a time. Skipping the details, this can be accomplished via a directional derivative

$$\frac{\partial}{\partial t}\chi(\phi, t) = \sum_{n=1}^{\infty} d^3x \dot{h}_{ab}(x,t) \frac{\partial}{\partial h_{ab}(x)}\chi(h_{ab}, \phi) \tag{6}$$

that we might call **WKB time**.

Notice what we have done. The approximations yield solutions of a classical-like general relativistic equation. We identified the natural time parameter in such solutions. Now we derive a **TDSE** using this time.

When we plugged our **WKB** approximation (4) into equation (1), we found that $S(h)$ satisfies the semiclassical classical gravitational equations up to leading order in m_p. Now we want to do the same but focus on $\chi(\phi, h)$. So we expand

$$\chi(\phi, h) = \psi(\phi, h) + \sum_{n=1}^{\infty} \left(m_p^{-2}\right)^n \psi_{(n)}(\phi, h) \tag{7}$$

and plug this into (1) using our **WKB** approximation $\Psi \approx A(h)e^{im_p^2 S(h)}\chi(\phi, h)$. Keeping only lowest order terms and doing a lot of massaging, we find that $\psi(\phi, h)$ satisfies a functional Schrödinger equation

$$i\frac{\partial}{\partial t}\psi(\phi, t; h) = \hat{H}^m(\phi; h)\psi(\phi, t; h) \tag{8}$$

where $\hat{H}$ is a Hamiltonian-type term and ψ is evaluated at a solution h_{ab}, which itself is a solution of the classical Einstein equations.

It is important to note that the "t" in (8) is the same as that in (6). The t we used to parametrize the classical general relativistic solutions (corresponding to the first "heavy" term in our factorization (3)) is used in a solution as a clock for the matter fields in the **WKB** regime. We won't delve into the rest of the theory; however, we should note that we can also derive a continuity equation that justifies the use of the normal Born rule for predictions from the theory, and furthermore using perturbation theory – by considering the higher-order terms we have so far ignored – one can derive non-classical predictions.

In sum, the semiclassical derivation provides an elegant derivation of time from no-time. Making a series of seemingly reasonable assumptions, a parameter that looks and acts like time emerges. And if we agree that something that looks and acts like time *is* time, then time emerges.

3 Justifying the Approximations

We jumped from one equation to another by expanding to leading order, focusing on lowest order, assuming the wave-functional approximately factorises, and so on. What justifies these steps? Approximations require a physical justification. At the level of pure math, one can "derive" virtually any equation from any other if one is allowed to assume anything. It makes no sense to say that one equation or quantity is "close" to another absent a metric. We need a justification, and it is in this physical justification that we believe time sneaks in.

To elaborate, we can treat a classical pendulum as approximately an undamped harmonic oscillator. That is because, for small angles, $sin\ \theta = \theta$, which allows us to derive an equation of motion for the pendulum that is the same as that of the harmonic oscillator. A harmonic oscillator, we might say, "emerges" from the pendulum in the small angle limit. But relative to some measurement standard, at some point an initial displacement angle becomes too big and the approximation fails; that is, we notice deviations from the derived equation of motion. Angles, theta, aren't intrinsically big or small. They are big or small relative to a standard. Typically that standard refers to the observational or measurement capacities of an observer. The validity of the approximation hangs in part on an error analysis of our measurement technique. Coarse measurements will allow the approximation to be good for greater values of θ than finer measurements.

This observation leads to a subtle problem for the semiclassical time program and also our analysis here. Since we don't have a fully interpreted theory yet, one with observers, we don't know what this standard will be. When are (say) off-diagonal terms in a matrix "small" and justifiably ignored? The answer is: when they're irrelevant to the observer (measurement, analysis, etc). But to introduce an observer to get the standard for judging small, we effectively need to already have a time. Observation is a temporal process, after all. So we can only justify an approximation by already introducing a time, making the derivation circular. And minus an observer, we can't say what "looks" like a small difference that would warrant an approximation.

We'll return to this point later, but for now we'll keep things simple. We'll point out that the approximations used to derive semiclassical time are always warranted in the rest of physics by appeal to an implicit time metric. Without the time metric, the approximations seem physically unwarranted. We do not, and cannot, show that there is *no standard possible that would warrant the approximations*. However, this point is worryingly suspicious and throws down a challenge to the advocate of semiclassical time: justify the approximations without appealing to a prior time standard.

Although the semiclassical time program has an estimated twenty assumptions (Anderson 2007), here we concentrate on three: the Born Oppenheimer approximation, the **WKB** approximation, and decoherence.

3.1 The Born-Oppenheimer Approximation

The usual story justifying the **BO** approximation in the semiclassical time program goes like this: we can think of the entire universe as containing two kinds of subsystems, the gravitational field h and the quantum matter fields ϕ. The masses associated with h are 'heavy' in comparison to ϕ (in the sense of being much larger than m_p),[2] and so it seems plausible that h is largely insensitive to ϕ. By contrast, ϕ, being small and light, is sensitive to the big and heavy h. We can therefore assume that the wave-functional Ψ for the entire system (the universe) can be approximately factorized into two wave-functions $\chi(h)$ and $\psi(\phi, h)$, with χ associated with the heavier h, and ψ associated with the lighter ϕ but also dependent on h, as in (3) above. This factorization, as we saw above, is a necessary assumption in the derivation above.

On its face, the assumption doesn't sneak time into the derivation. Some masses are larger than others, and we expand accordingly. "Change" above needn't be temporal change.

But it might be. The **BO** approximation is motivated by an appeal to the "very different scales" (Kiefer 2004, 164) that the gravitational fields and matter fields have. This appeals to a metric that measures how big the effects of the one subsystem are on

[2]See e.g. Banks (1985, 337–338), Kiefer (2004, 165).

the other. Why does having different size masses warrant different scales? Differences in other properties, say charge, do not, so why mass? We must probe deeper.

To help, let's review standard uses of the **BO** approximation. Since it is so widely and successfully used, it may be imported into a derivation without considering whether the conditions warrant its use in a new application. Unfortunately we'll find that mass and size scale differences between systems are only relevant for the **BO** approximation because they are proxies for *timescale differences* in the dynamics of the relevant subsystems.

In its most popular application – molecular and atomic physics – the **BO** approximation is used to factorize an atom or molecule's wave-function into the product of two subsystems. Here, the heavier subsystem is the nuclei, and the lighter subsystem is the electrons surrounding the nuclei (Griffiths 2005). Again, the heavier system is assumed to be effectively independent of the lighter system, while the lighter system rapidly adapts itself to changes in the heavier system. The usual procedure in these contexts is to pretend that the nuclear wave-function is not changing at all *in time*, and then calculating the electronic wave-function associated with that nuclear wave-function. We then calculate a more realistic nuclear wave-function by letting it vary 'slowly' or 'sluggishly', calculating the possible ranges of electronic wave-functions and hence the mean potentials in which the nuclei can move.

More generally, the **BO** approximation applies in cases where heavier subsystems are known to change slowly *in time* with respect to lighter subsystems. In these cases, heavier subsystems have significantly different *characteristic dynamical timescales* – timescales over which "the parameters of the system change appreciably" – with respect to lighter subsystems, and can be said to be *adiabatic* with respect to the lighter subsystem. The change in the lighter subsystem happens on such a short timescale that there isn't enough time for the heavier subsystem to react in that relevant timescale, and so it is effectively independent of lighter subsystems in that period of time - hence the **BO** approximation. The **BO** approximation is thoroughly laden with temporal notions.

Returning to the semiclassical time program, we have a dilemma at hand: either the mass scales relevant here are proxies for time scales or not. If they are then we face circularity; if they are not, then we have no clear means of assessing whether **BO** is even applicable in this situation. In short, this seems to be a case of needing time to get time, but of course, we have no time for that in the semiclassical time program.

*3.2 The **WKB** Approximation*

The **WKB** approximation is a staple of every quantum mechanics course. Because it is often presented as a piece of pure math, this makes **WKB** seem like it is simply an approximation method in the theory of partial differential equations, an unlikely place to find a hidden time preference. But of course, we still need physical justifications for why this math applies to a given physical situation. For that we need physics.

Frequently **WKB** is used when one is working with stationary states of energy $E > V$. We note that right away the time dependence is therefore hidden. If a system begins in an energy eigenstate with eigenvalue E, then time evolution simply multiplies the state by a time-dependent phase factor that does not affect the probabilities for measurement.

However, this is not to say that the approximation does not sneak in time elsewhere. We can see this most clearly with the textbook **WKB** derivation. Let us begin with the one-dimensional **TISE** describing a system in a background potential $V(x)$:

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - V(x))\psi = 0 \tag{9}$$

or:

$$\frac{d^2\psi}{dx^2} + \frac{p(x)^2}{\hbar^2}\psi = 0 \tag{10}$$

where we use the classical momentum identity:

$$p(x) = \sqrt{2m(E - V(x))} \tag{11}$$

If $V(x)$ is constant, then the system behaves like a free particle with $\psi(x) \sim e^{ip(x)}$. If $V(x)$ varies *slowly*, then we might expect that the system behaves *approximately* like a free particle. Motivated by this thought, we look for solutions to the **TISE** of the form

$$\psi(x) = A(x)e^{iS(x)/\hbar} \tag{12}$$

Plugging this back into the **TISE**, we get two equations (for the imaginary and real parts respectively):

$$\hbar\frac{d^2A}{dx^2} = A\left((\frac{dS}{dx})^2 - \frac{p(x)^2}{\hbar^2}\right) \tag{13}$$

$$2\frac{dA}{dx}\frac{dS}{dx} + A\frac{d^2S}{dx^2} = 0 \tag{14}$$

Everything so far is exact. However, it is important to note that (13) *generally does not have analytic solutions*. What then? The solution, and a crucial step in the **WKB** approximation, is to *assume* that A varies *so slowly* with respect to x that $\frac{d^2A}{dx^2} \approx 0$.[3]

This crucial step allows us to solve (13) and (14) for A and S. Combining these results with (15), we get to the well-known **WKB** approximation to the wave-function

[3]More precisely, as Griffiths (2005, 317) notes, we assume that $\frac{d^2A/dx^2}{A} \ll (\frac{dS}{dx})^2$ and $\frac{d^2A/dx^2}{A} \ll \frac{p(x)^2}{\hbar^2}$. This step is also equivalent to taking only the zeroth-order term when expanding S in orders of $\hbar$, a move we have already seen in (5) above.

$$\psi(x) \approx \frac{A}{\sqrt{p}} exp\left(\pm \frac{i}{\hbar}\int dx\ p(x)\right) \tag{15}$$

Arbitrary superpositions of these wave-functions are approximate solutions of the Schrödinger equation. They are also exact solutions of the classical Hamilton-Jacobi equation — from which one obtains the time parameter used in the semiclassical time program.

Under what conditions are we allowed to neglect $\frac{d^2A}{dx^2}$? This is where the physics comes in. The answer is well-known: V must vary slowly with x and $(E - V)$ is not too small. When V is constant, and the system behaves like a free particle, A is constant. When V is 'close to constant', i.e. varying slowly, so too is A.

On its face, the condition of V "slowly varying" does not conceal any time-dependence, since we are only concerned with slowness with respect to the spatial x, not the temporal t. What seems to motivate **WKB** is that when the potential is not too spatially sharp one tends to not see much interference, so this important assumption is about spatial smoothness not temporal variation.

Still, time is very much present. There are probably many ways to see this. One is to note that if the potential spatially varies slowly with respect to the de Broglie wavelength of the particle, then the wave-function will approximate that of a free particle, i.e., a plane wave. That means the system will propagate freely with a constant velocity v for a time T. As Allori and Zanghi (2009, 24) note, that time – the time for which we can pretend that V is effectively constant – has the following relation:

$$T \sim \frac{L}{v} \tag{16}$$

where L is the scale of the variation of the potential.[4] This gives us a clear physical picture of what it means to apply **WKB** in the first place. If L is long and v is low, then the particle is moving slowly through an effectively unchanging V, allowing **WKB** to hold for long times. Conversely, if L is short and v high, then the particle rapidly moves (in time!) through the potential – in these cases we can no longer assume that V is effectively constant for the system, and **WKB** will not hold for long times. Because $\lambda = \frac{\hbar}{p} = \frac{\hbar}{mv}$, we can write (21) as:

$$T \sim \frac{Lm\lambda}{\hbar} \tag{17}$$

The time-dependence, evident when talking about velocities, becomes masked when we replace velocities with purely spatial notions of wavelengths and spatial variations. Yet, the time-dependence is very much still there as we can see in (21), and all the physical

[4] Allori and Zanghi (2009, 24) use the following examples for L: $L = \infty$ for a constant potential, and $L = a$ for a potential of the form $V(x) = sin(\frac{2\pi}{a}x)$.

reasoning happens there. And, from (21) and (22), we can see that if L is large, the **WKB** approximation will be good for long T and if small then only for short T. Obviously, if **WKB** only holds for vanishing T, then it is physically unrealistic to apply it. On the contrary, if it holds for large T, then it is physically warranted.

In standard cases **WKB** is thus justified via a background time metric. In the case of semiclassical time, however, there is no such background time metric, so again, we have a challenge: why do the spatial scales (in λ and L) justify **WKB**? We know of no relevant metric in the timeless scenario that will work.

3.3 Decoherence

The discerning reader might have noticed two sleights of hand in the derivation of the functional **TDSE** and 't'. First, in the **BO** approximation, we effectively assumed that Ψ was an eigenstate (3) of the **WD** equation. Since the **WD** equation is linear, a general solution will involve a superposition of states. Second, a similar assumption was made in our choice of the approximate **WKB** wave-function for the gravitational fields $\chi(h)$ (4). Since the **WD** equation is also linear, arbitrary superpositions of states are again also solutions. These assumptions are absolutely vital for deriving a functional **TDSE**.[5] Using an arbitrary superposition of states in the **BO** and **WKB** approximations, the above procedures do *not* recover a semiclassical time.

The most popular response to these observations appeals to *decoherence*, e.g., Kiefer (2005, 317). The idea is that if the initial state of the universe is in an arbitrary superposition of states, then decoherence will drive the wavefunction into a superposition of effectively non-interacting components, each one of which is suitable for the semiclassical time recovery. In an Everett-type interpretation of quantum mechanics, for instance, we could recover a time in each decohered branch or world.

Our worry is especially clear in this case because decoherence is normally understood as a dynamic process. It presumes temporal evolution by the Schrödinger equation. Decoherence at once requires time and is required for time. Indeed, one can find tension in Kiefer's own account. On the one hand, he writes that "A prerequisite [of decoherence in the semiclassical time program] is the validity of the semiclassical approximation (Section 5.4) for the global variables. This brings an approximate time parameter t into play." (Kiefer 2005, 311) But later he writes that "Since [decoherence] is a prerequisite for the derivation of the Schrödinger equation, one might even say that time (the WKB time parameter in the Schrödinger equation) arises from symmetry breaking [i.e decoherence]... Strictly speaking, the very concept of time makes sense only after decoherence has occurred." (Kiefer 2005, 318) Obviously, the two claims cannot be true at once, and again, we face our dilemma.

[5]See Kuchar 1992.

4 Discussion

Overall, our investigations into three approximations integral to the semiclassical time approach have unearthed a general worry: we seem to need to put time in, somewhere and somehow, in order to get time out of the timeless formalism. We haven't shown that there is no possible way to do this. If we could make sense of an atemporal observer, for instance, perhaps we could show that relative to her standards of measurement, the terms ignored in **BO**, decoherence and **WKB** are in some sense small. Even if we could do this, however, there is very little to work with in canonical quantum gravity to help us make justify these approximations. This point will become clear if we compare our objections to a very similar one leveled against decision-theoretic attempts to derive Born's rule in Everettian quantum mechanics.

As is well-known, the Everett interpretation faces a problem in making sense of the probabilities we use in quantum mechanics. Its laws consist only in a linear deterministic wave equation. Therefore it produces only trivial probabilities (0, 1) for any outcome. Born's Rule, our guide to experiment, seems unexplained. In response, one school of Everettian thought turns to decision theory. The idea is to prove that an Everettian agent, if she is rational, will set her preferences in accord with Born's Rule. Controversy ensues about whether the assumptions used in the proofs are really requirements of rationality.

But there is another line of criticism that will immediately sound familiar. Baker (2007), Kent (2010) and Zurek (2005) all point out that Everettians use decoherence to say that different "worlds" approximately emerge from the wave-function. What does "approximately" mean here? Well, it seems to mean that a branching structure is likely to happen, i.e., that the probability of an error is small according to the Born measure (mod-squared amplitude). Yet all the decision theoretic proofs begin with a branching structure. That begs the question, the critics say, for we've assumed that mod-squared amplitude is a probability in our demonstration that mod-squared amplitude is probability.

Structurally this objection is very similar to ours. Can any replies in the probability case be transferred to the present one?

The only response on behalf of the Everettian that we have been able to find is Wallace (2012, 253–4). As we understand him, Wallace argues that the branching structure "really is robustly present" even prior to the interpretation of mod-squared amplitude as probability. What standard makes it present? His answer: Hilbert space norm. This is an objective physical measure. If branching emerges approximately with respect to Hilbert norm, then the probability measure is not needed as an assumption assumption in the derivation of Born's rule. One could fairly ask whether Hilbert space norm is really enough to answer the objection. Small differences in Hilbert space norm may not be small differences for an observer, or vice versa. From color science we know that colors that look similar (have small phenomenological distance) might be produced by physically quite dissimilar properties. The Hilbert space norm might not be enough for Wallace to fully answer the charge.

However that debate gets resolved, we want to note that in the present case we lack anything like Hilbert space norm. The space of all spatial three-metrics has a geometry to it, given by the so-called DeWitt metric. But this metric won't tell us how far quantum states are from one another. What we would need, comparable to the Hilbert norm, is an invariant positive-definite inner product on the space of solutions of **WD**. But here we're right back to time! "Invariant" means that the inner product is independent of time. Constructing an invariant positive-definite inner product on the solution space of the **WD** equation is the notorious "Hilbert space problem" (Kuchar (1992). Whereas the Schrödinger equation brings "for free" a nice conserved inner product, **WD** does no such thing. The most natural way to solve the Hilbert space problem is to identify a time variable and construct a norm from that; but of course in this context that won't help.

Lacking observers, we don't want to say that there is no atemporal metric available to warrant the approximations. We haven't shown that. But we have shown that the most natural warrant – and the warrant found hidden throughout the use of our assumptions – is temporal. And we see no reason to think the introduction of observers will change that verdict.

5 Conclusion

We started with the core idea that the world was fundamentally timeless: semiclassical time arises from certain regimes looking temporal when we blur our vision. That metaphor turns out to be not quite right, as it neglects that we've imported a mathematical construct, the Hamilton-Jacobi structure, onto the basic physics and only within that structure does something naturally corresponding to time emerge. Instead of blurry vision making a pattern of correlations in the wave-functions look temporal, what's really happened is that we're being offered "time glasses." We are told that you're justified in using these glasses – this mathematical construct – and when we look through them, they turn the pattern temporal. But are we justified in wearing "time glasses"? At present, it seems that the only reason to wear them is when one already has time.

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