ELSEVIER

20 November 1995

PHYSICS LETTERS A

Physics Letters A 208 (1995) 59–61

Bohmian cosmology and the quantum smearing of the initial singularity

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Received 1 May 1995; accepted for publication 11 September 1995
Communicated by P.R. Holland

Abstract

We examine a recent quantum cosmological model by Smith and Bergmann from the point of view of Bohm's interpretation of quantum mechanics. We show that the quantum smearing of the initial singularity in this model does not prevent the formation of a singularity if interpreted on the Bohmian model. Furthermore, it is argued that from this perspective, the criterion used for establishing the existence or nonexistence of quantum singularities is fundamentally misconceived.

In a recent article Smith and Bergmann [1] present an interesting quantum cosmological model. In this model the quantized Wheeler–DeWitt equation has the form

$$H\Psi = (m_P hN - \gamma)\Psi = 0, \tag{1}$$

where H is the Hamiltonian, m_P the Planck mass, N the number operator, and γ the energy of incoherent electromagnetic radiation. This is just the one-dimensional harmonic oscillator, with solutions ψ,

$$\psi = |n\rangle,$$

labeled by eigenvalues n of N, and the allowed values of $\gamma = m_P hn$. The novel idea of the model is that the radius R of the universe is not a dynamical degree of freedom. Rather, a new variable ρ is introduced, such that

$$R = |\rho|,$$

ρ is the dynamical variable in the coordinate representation of (1). Since the range of ρ is $-\infty < \rho < \infty$, all the solutions $|n\rangle$ are available, instead of just the ones odd in ρ, which is what would be the case if R were the dynamical variable with range $0 \leqslant R < +\infty$. Because of this, Smith and Bergmann are able to base their model on the coherent state ψ_c,

$$\psi_c = H \exp[\gamma(t)a^+]\,|0\rangle, \quad \gamma(t) = \gamma\exp(-it),$$

$$H = \text{const},$$

which is obviously a solution of (1) since $|n, \gamma\rangle$ is a solution for each n, γ and $\psi_c(t)$ has the form $\sum A_n|n, \gamma\rangle \exp(-inwt)$, with $w = m_P h$.

But what is the meaning of the variable "t"? In terms of a time-dependent Schrödinger equation, we have

$$ih\, d\psi_c/dt^* = H\psi_c = 0, \tag{2}$$

which seems to imply that any ψ which satisfies $H\psi = 0$ must be time-independent. However, that is wrong. What follows is that ψ must be t^* indepen-

SSDI 0375-9601(95)00701-6

dent. The hypothesis implicit in the Smith–Bergmann model is that while H is the generator of t^* translations, t^* parameters are *not* physical time variables for the universe. That is played by the t variable in ψ_c. Certainly, one can question the status of the t time. In our opinion it is best viewed as having a theoretical status, rather like the absolute time of Newtonian dynamics. In any case, what is interesting about this model is that if we do identify t with the physical time, then we have a smearing of the "initial" singularity in the sense that while

$$\langle \rho \rangle_c = \sqrt{2}\,\gamma \cos t$$

when ρ is squared,

$$\langle \rho^2 \rangle_c = \langle R^2 \rangle_c = 2\gamma^2 \cos^2 t + 1/2.$$

At values of t which are zero of $\langle \rho \rangle_c$, $\langle R^2 \rangle = 1/2$. If the classical metric is identified, as is usual with coherent states, with the expectation values $\langle g_{\mu\nu} \rangle_c$ that are proportional to $\langle R^2 \rangle_c$, then we see that the spacetime metric is well-defined even at a zero of $\langle \rho \rangle_c$ which, presumably, is an infinite density singularity.

However, it is no secret that many people, including us, think that the standard interpretation of quantum mechanics is plagued by conceptual difficulties. One way of resolving these difficulties is Bohm's 1952 interpretation (see Refs. [2,3]). In this note we wish to look at the Smith–Bergmann model from the point of view of Bohm's theory.

To begin with, we try to follow our earlier Bohmian interpretation of quantum cosmology [4] (but also see Ref. [3], pp. 567–571). The idea in the present context is that if $H\psi = 0$, then we still get a time-dependent ρ via the Bohm equation of motion, which generically would be

$$\mathrm{d}\psi/\mathrm{d}t \propto \mathrm{d}S/\mathrm{d}\rho, \quad \psi = R(\rho)\exp[\mathrm{i}S(\rho)]. \qquad (3)$$

However, the time variable t^* in this equation is the same t^* that appears in the time-dependent Schrödinger equation (2), as is obvious from the derivation of (3). Substitution of $\psi = R\exp(\mathrm{i}S)$ into the Schrödinger equation, and separating into real and imaginary equations, the real equation can be interpreted as a generalized Hamilton–Jacobi equation and this yields (3). According to this Bohmian interpretation then, the t variable in the Smith–Bergmann model is not the physical time with respect to which ρ evolves. The variable t is therefore not physical and has no justification.

Even so, there is a more fundamental point to be made. Namely, even if the Bohm time t^* did coincide with the Smith–Bergmann time t, it would still be true that $R = |\rho|$ and therefore, according to a Bohmian interpretation, R^2 always has a definite value. This is because the new dynamical variable ρ plays the role of the particle position in Bohm's theory, and particles in this interpretation always have well-defined trajectories. We do not need $\langle R^2 \rangle_c$ to play the role of R^2, and indeed it cannot in the theory any more than $\langle x^2 \rangle$ can play the role of x^2 in Bohm's original theory. Therefore, on a Bohmian interpretation, $\langle R^2 \rangle_c$ is irrelevant to the behavior of the universe at the singularity. If $R = 0$ is an infinite density and curvature singularity, then R^2 is not well-defined, no matter that $\langle R^2 \rangle_c = 1/2$! Thus, one popular way of having quantum mechanics circumvent a classical singularity is shown to be fundamentally misconceived from the perspective of Bohm's theory. The problem ultimately lay in the criterion for the existence or nonexistence of a singularity. A singularity is judged to exist when the expectation values for the operators associated with the classical quantities which vanish at the classical singularity also vanish [5,6]. But we have seen that in Bohm's theory the expectation values may be nonsingular even when the radius of the universe does vanish. This may suggest that the quantum criterion for singularities may not be a reliable guide to their genuine existence or nonexistence.

Appendix

For the sake of completeness, we give $\rho(t^*)$ and $\rho(t)$.

(A) Using the Bohm time t^*, we look at the coherent state without the Smith–Bergmann time t: then

$$\psi_c = H' \exp(-\rho^2/2 - \gamma\rho)$$

in the ρ representation. If w is real we get no evolution of ρ, but if γ is complex, $\gamma = a\mathrm{i} + b$, the phase $S = b\rho$ and

$$\mathrm{d}S/\mathrm{d}\rho = \mathrm{d}\rho/\mathrm{d}t^*$$

implies $\rho = bt^* + \rho_0$.

(B) We momentarily assume that the time t of the Smith–Bergmann model has a sound quantum theoretical justification of some kind and apply Bohm's formula to it. With $\gamma(t^*) = \gamma \exp(-\mathrm{i}t)$, γ real,

$$\psi_c(t) = H \exp\left[-(\rho^2/2 - \gamma\, \mathrm{e}^{-\mathrm{i}t}\rho)\right.$$

$$= H \exp(-\rho^2/2) \exp[\gamma\rho(\cos t - i \sin t)]$$

so the phase $S = -w\rho \sin t$ and

$$\rho = \gamma \cos t + \rho_{(t=\pi/2)}.$$

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