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Source: *PSA: Proceedings of the Biennial Meeting of the Philosophy of Science Association*, Vol. 1994, Volume One: Contributed Papers (1994), pp. 218-227
Published by: The University of Chicago Press on behalf of the Philosophy of Science Association
Stable URL: http://www.jstor.org/stable/193027
Accessed: 08/05/2014 23:03

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1. Introduction

Philosophers of science have not paid much attention to recent developments in quantum cosmology. This fact is surprising, since quantum cosmology is replete with conceptual issues involving (e.g.) the fundamental nature of time and space, the interpretation of quantum mechanics, and the ultimate meaning of probability. One notable exception, Quentin Smith, has recently examined the Hartle-Hawking (1983) proposal. Trying to make sense of the view, he resorts to an instrumentalist picture, which treats the proposal as merely a heuristic device for the algorithm responsible for predictions. While we do not examine Smith's account here, we would like to contrast it with the model presented in this note, in which a fully realistic interpretation of quantum cosmology is developed.

Recently there has been a resurgence of interest in the de Broglie-Bohm causal interpretation of quantum mechanics. The merits of this interpretation regarding non-relativistic quantum mechanics are extolled elsewhere, and shall not be repeated here (see Albert 1993, Bell 1987, Bohm and Hiley 1993, Durr et al 1992). The present essay concerns the relationship between Bohmian mechanics and recent problems in quantum cosmology. We argue that when cosmological factors are considered, the de Broglie-Bohm interpretation remains the only satisfactory interpretation of quantum theory. This assertion is advanced with a Bohmian resolution of (one aspect of) the so-called problem of time in quantum cosmology. Moreover, the preceding is accomplished without having to split worlds, multiply minds, or ever worry about observers collapsing wavefunctions.

2. Bohmian mechanics

Nonrelativistic Bohmian mechanics is characterized by two basic equations of motion. One governs the wave function $\Psi = A\exp[iS(x)]$, the other the particles postulated by the theory. It is convenient to rewrite the Schrodinger equation as a modified Hamilton-Jacobi equation

$$dS/dt + (\nabla S)^2/2m + V + Q = 0 \qquad (1)$$

where Q, the so-called quantum potential, is given by

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$$Q = -h^2\nabla^2 R/2mR.$$

(1) continues to be the equation of motion for the wave function. As in ordinary Hamilton-Jacobi theory, probability is conserved for particles satisfying (1), provided the particles have momentum $p = \nabla S$. This feature produces an equation for the particles:

$$m dx/dt = -\nabla(V) - \nabla(Q). \quad (2)$$

On the assumption that the probability density equals $|\Psi|^2$, Bohmian mechanics reproduces the results of ordinary quantum mechanics. The theory is deterministic, and provides a conceptually clear account of quantum phenomena; in particular, it does not suffer from a problem of measurement (see Bohm and Hiley 1993).

3. The Problem(s) of Time in Quantum Cosmology

The 'problem of time' in canonical quantum gravity (QG) and quantum cosmology seems not to refer to a particular problem, but to an ill-defined set of related problems. Time in quantum mechanics is essentially the immutable, external time of Newton, whereas general relativity treats it as an arbitrary parameter. This incompatibility creates a multitude of difficulties, e.g. the factor-ordering problem, the problem of observables, the Hilbert space problem (see Kuchar 1992 for an excellent review). No one of these is properly singled out as *the* problem of time.

Even so, from the perspective of cosmology one problem is especially vexing. In QG the Hamiltonian for the universe is identically zero. Vanishing Hamiltonians are not generally problematic, for time variables can usually be physically identified through a system's interaction with other systems. But when the entire universe is the subject, the vanishing of the Hamiltonian does present a serious difficulty.

At the very least, the vanishing of the Hamiltonian apparently spoils the desired interpretation of the principal equation of QG, the Wheeler-DeWitt equation

$$H\Psi_{wd} = [{}^1\!/_2 G_{abcd} p^{ab} p^{cd} - |h|^1\!/_2 R + H_{matter}]\Psi_{wd} = 0. \quad (3)$$

Here G is the DeWitt metric, representing the intrinsic geometry of a point in 'superspace.' Superspace is the configuration space consisting of the set of equivalence classes of Riemannian metrics on (usually compact) spatial 3-geometries. p^{ab} is the canonical momenta conjugate to h_{ab}, $|h|$ the volume element of the 3-space h_{ab} and R its scalar curvature. The picture sought in QG is one in which a 3-geometry evolves in superspace along an arbitrary time parameter τ. In the naive interpretation, Ψ is intended to provide the probability amplitude for a particular 3-space obtaining at a time parameter τ. Since $H = 0$, the wave function of the universe is independent of time. Hence the 3-space does not evolve in time. If the 3-space is considered analogous to a particle, it is in an eigenstate of zero energy, whose state cannot be affected by any measurement. Not only doesn't the universe described by (3) expand, then, but contrary to our experience, it appears utterly static. One often hears the possibly exaggerated claim that time 'disappears' in QG.

4. Interpretations of Quantum Mechanics

If the situation in cosmology is used as a yardstick by which interpretations of quantum theory are measured, the de Broglie-Bohm model seems uniquely fit. First, unlike the orthodox interpretation and its variants, it requires no external measuring apparatus to reduce superposed wave functions. Since cosmology is concerned with the wave function of the entire universe, it makes little sense to speak of external measurements collapsing the wave function. Even on 'collapse' versions not requiring measurement, such as Ghirardi et al (1986), to speak of the collapse of the universe's wave function is awkward at best, given the aforementioned static situation. Second, the main reason to quantize gravity is that Einstein's field equations look inconsistent, since they set an ordinary function of spacetime points equal to a quantity depending on quantum operators. Since in Bohm's theory the operator-formalism emerges as only a phenomenal (measurement) description of the underlying physics, this prima facie conflict doesn't signify any problem deeper than that. Third, a little-mentioned point is that in cosmology most 'predictions' are really retrodictions. Bohmian mechanics, in contrast to most interpretations, allows us to know more about the past than the future. Integrating the velocity field equation backwards in time allows for accurate retrodictions to be made. And this is the case even if the initial wave function is a superposition of states.[1]

For these reasons, Bohm's interpretation seems well suited for cosmological application. It is also preferable to the interpretations presently used in quantum cosmology, versions of the many-worlds and many-minds interpretations (see Albert and Loewer 1988 for discussion of both and references). In our opinion, though explanatorily useful, these two views are utterly fantastic. It is simply incredible that quantum probability distributions are given by the 'trajectories' of the continuous infinity of minds associated with each observer, or by the splitting of worlds or 'relative states' (whatever they may be). That so many have swallowed such notions is remarkable.

Because the many-worlds interpretation is so popular, a few remarks should particularly be directed its way. (1) Decoherence may solve the so-called 'basis problem,' but it does not explain in what sense many-worlds describes a probabilistic theory. Each measurement outcome in our universe has probability one, since the entire universe corresponds to one ray in Hilbert space. (2) The meaning of the wave function is muddled in many-worlds. The range of quantum mechanics is ambiguous: is it a theory of our world, or of the continuous infinity of worlds? (3) Unlike in Bohm's theory, wherein the classical limit could not be any clearer ($Q \longrightarrow 0$), the classical limit in most presentations of many-worlds is mysterious.[2] Further critical discussion of these issues can be found in Albert and Loewer (1988) and Bohm and Hiley (1993). When these problems are contrasted to the clarity of Bohmian mechanics, and its resolution of the problem of time (in section 7), we believe a compelling case is made for the superiority of the causal interpretation.

Finally, it has been asserted that many-worlds and/or many-minds can solve the problem of time in quantum cosmology (Squires and Collins 1993). We wish to dispute this claim. It is true that projection operators can be introduced which do not commute with the Hamiltonian, thereby producing non-trivial time evolution. But the *physical meaning* of these projection operators needs to be clarified. Why is the wave function projected, and what selects the associated eigenvalues? It is not surprising that advocates of these theories gloss these questions, for good answers are not forthcoming. If consciousness is involved in the answers, for instance, then the universe did not start evolving until conscious minds 'entered' the universe. The only way to make sense of this result is to claim the universe began with conscious minds in it, and that the appearance of prior development is illusory. Surely cosmologists can't be happy with that. If 'something else' is involved in the answers, we wish to know what it is. Until then, the purely formal application of projection operators to regain time is not physically justified.

5. The Klein-Gordon Analogy

Since QG is so poorly understood, one usually works with simple models which are described by equations sharing significant features with the Wheeler-DeWitt equation. The model used here is the relativistic particle described by the Klein-Gordon (KG) equation. To mirror the Wheeler-DeWitt equation (3) the Hamiltonian of the KG equation is turned into an operator, and imposed as a restriction on the space of physical states:

$$H\Psi_{kg} = [\tfrac{1}{2}G^{ab}P_aP_b + \tfrac{1}{2}MV]\Psi_{kg} = 0. \tag{4}$$

The analogy with (3) is a good one: the position plays the role of the intrinsic geometry, the background metric the role of the DeWitt metric, and the potential the role of the scalar curvature. Also, the kinetic portion of each Hamiltonian is indefinite (the geometrodynamical potential is also indefinite, whereas the relativistic particle's is positive definite). In fact, the relationship is better than an analogy: when (3) is restricted to two degrees of freedom, it *is* (4).

Notoriously, the KG equation suffers from a serious problem. Its inner product $<\Psi_1|\Psi_2>$ is not positive definite, and therefore cannot be used to define a probability. A single relativistic particle has no suitable Hilbert space.[3] The traditional response to this problem is to rewrite the theory as a field theory on Fock space. States of arbitrary particle number are allowed, and the wave function is second quantized. In second quantization, the 'wave function' $\Psi(x)$ becomes a field operator $\hat{\Psi}(x)$, which acts on the Hilbert space of states. The wave function of the field, however, is a functional of the field configuration $\omega(\Psi(x))$ — in that representation.

In QG the states Ψ are functionals of the metric $\Psi(h(x))$. If we were to 'third' quantize, then the Ψ becomes an operator $\hat{\Psi}(h(x))$ on states, and the wave function a functional of the field $\Psi(h(x))$ — $\omega(\psi(h(x)))$.

6. Bohmian Third Quantization

Even if quantizing $\Psi(h(x))$ to obtain the operator valued field $\hat{\Psi}(h(x))$ in superspace solved the Hilbert space problem, we still have the problem of the interpretation of quantum mechanics. So we would like to apply Bohm's theory to $\Psi(h(x))$ as well. Since Bohmian particle mechanics has been extended to bosonic field theory, it is possible to investigate a Bohmian version of third quantized QG.

Bohmian field theory is formulated in terms of a super-wave $\Psi\,[\phi(x)]$ over a full field configuration. The characteristic feature of Bohmian mechanics, the quantum potential, has a field theoretic analogue derived from the Schrodinger equation

$$Q[\phi(x)] = -\tfrac{1}{2}h^2\int d^3x\ (\partial^2R[\phi]/\partial\phi^2(x,t))/R[\phi].$$

$Q[\phi(x)]$ modifies the KG equation, and guarantees that Bohm's causal field agrees with predictions made by conventional quantum field theory. In Bohmian field theory $\Psi[\phi(x)]$ is interpreted as the probability amplitude for finding field values $\phi(x)$ when the system is in state Ψ. Further details about this approach can be found in Bohm and Hiley (1993), Bell (1987) and Huggett and Weingard (1994).

In Bohmian third quantization, $\Psi(h(x))$ becomes a 'physical' field, the beables of the theory, on analogy with $\phi(x)$ when Bohm's theory is applied to the scalar field. In the case of the Bohmian scalar field, there are 0-particle, 1-particle ... n-particle field configurations, but these are just ϕ-field configurations that interact with localized measuring devices *as if* 0,1,...n 'localized' discrete entities (with continuous trajectories) were present. But there are no such entities present, there are only ϕ-field configurations.

What about the similiar situation for $\psi(h(x))$ on the analogy between $\phi(x)$ and $\psi(h(x))$? The configuration space for $\psi(h(x))$, analogous to x for $\phi(x)$, is the space — superspace — of 3-metrics h(x). The space of h(x) doesn't, presumably, have the ontological status of the configuration space x, but Bohm's theory does assign a definite value to ψ for each 3-metric h(x) (given suitable boundary conditions). Let's emphasize, superspace is a configuration space, relative to which values of ψ are defined; its points are not the values of actual universes, any more than the x's in the $\phi(x)$ are actual values of the positions of the particles. Now, just as in the scalar field, if the wave function is appropriate, then $\psi(h(x))$ will be a 0,1,...n universe field configuration. But if the analogy with the scalar field is good, and the field is the 'beable,' then there aren't any 'universes.' Consequently, the theory is scarcely comprehensible.

If third quantization solved the problem of time, it would warrant further speculation about its meaning. However, there is reason to think the problem survives third quantization, for second quantization itself is a casualty of the problem. As Kuchar (1992) has emphasized, quantum field theory on a dynamical manifold suffers from its inability to define a one-particle Hilbert space. Quantum field theory on a flat background is well-defined because the background admits the relevant isometries for construction of a one-particle Hilbert space (spanned by the positive energy solutions of the KG equation). This space is subsequently used to construct the Fock space. On a dynamic background, Hilbert spaces for stationary pasts and futures can be designed; however, it is not clear whether they can be built for the dynamical region (see fn. 3). Hence, when applied to dynamic backgrounds, second quantization itself apparently suffers from the problem of time. Consequently, it is unlikely that third quantization, which merely carries out the second quantization procedure, is going to do any better.

7. The Reappearance of Time in Bohmian Cosmology

Let's start again. This time we'll be less ambitious, and approach the Wheeler-DeWitt equation from a naive Bohmian perspective.

Consider a wave function where $\Psi = A\exp[iS]$. To keep matters simple, let Ψ be a function over just two variables, χ, the radius of the universe, and θ, a spatially constant scalar field. Substitute Ψ into equation (4). Separated into real and imaginary parts, (4) becomes

$$n^{\mu\nu}\partial_\mu S \partial_\nu S A - V - Q = 0 \qquad (5)$$

$$A^{-1} n^{\mu\nu}\partial_\mu(A^\nu \partial_\nu S) = 0. \qquad (6)$$

(5) is the Hamilton-Jacobi equation modified by the quantum potential (with E = 0). (6) is a continuity equation. The quantum potential takes the form

$$Q = -(\partial_\chi \partial^\chi A - \partial_\theta \partial^\theta A).$$

Continuing as in regular Bohmian mechanics, trajectories for the dynamical variables are obtained.

Therefore,

$$d\chi/dt = dS\chi(\chi,\theta);\ \ d\theta/dt = dS_\theta(\chi,\theta). \tag{7}$$

Dynamics are obtained with respect to a time parameter t. (We discuss the nature of this time in section 9). Contrary to the static situation described at the outset, a Bohmian approach to cosmology admits nontrivial evolution of the dynamical variables. The importance of the trajectories can not be underestimated. They allow for both dynamics and a clear interpretation. The former makes the theory physically viable, the latter makes it comprehensible (unlike most other quantum cosmological schemes). Since the temporal evolution of cosmological parameters can be predicted (and retrodicted) it amounts to precisely the picture wanted in cosmology.

Given such success from a relatively modest point, the reader may have the feeling that we have pulled a rabbit out of a hat. Where do the time and dynamics originate? The answer is that they arise from the laws of Bohmian mechanics. Unlike most interpretations of quantum mechanics, Bohmian mechanics is explicitly a theory with two fundamental equations of motion. One governs Ψ, the other the dynamical variables of the theory. In particle mechanics, the variable is position, in field theory the full field configuration, and now in cosmology, variables such as χ,θ. In short, the 'magic' of the result ultimately stems from the interpretation's recognition that two equations of motion are needed. Indeed, from a Bohmian perspective, it is not terribly surprising that quantum cosmology cannot describe a dynamic universe when only one of these equations is used.

8. Discussion

The first item to notice is that the calculation essentially has been done before, in the WKB interpretation of Ψ (see Halliwell 1991). The difference between the approaches lies in the drastically different interpretations of the formal result. The WKB interpretation tries to extract a probabilistic interpretation of Ψ, but only when $\Psi = A\exp[iS]$. If $\Psi = A\exp[iS]$ then the conserved current $j^A = |A|^2\nabla S$ can provide probabilities for observing classical trajectories. The interpretation states that if $\Psi = A\exp[iS]$ then there is a particular probability associated with measuring specific values of the dynamical variables. The interpretation suffers, however, because it is valid over only a very limited range, and generally, there is no reason to suppose the probability density will be nonnegative.

In our interpretation all wavefunctions permit the derivation of real deterministic trajectories. It is therefore clear and globally applicable. We do not extract probabilities from the KG equation (so it doesn't suffer from a Hilbert space problem). However, the Schrodinger equation still provides probabilistic results for quantum mechanics, thus emphasizing quantum mechanics' status as a measurement formalism. As in Bohmian mechanics these probabilities merely reflect our ignorance, not the underlying reality. That they are not objectively probabilistic is a significant virtue, in our opinion, for we wonder what it means for the entire universe to have a certain chance. Given the negligible formal difference between the WKB and causal interpretations, then, and the tremendous difference in clarity and application, we believe proponents of the WKB approach would do well to embrace our model.

The wave function employed is section 7 is complex, as in the Vilenkin (1989) model. If the wave function were real, like the Hartle-Hawking (1983) wave function, the dynamical variables would be at rest. As pointed out by Squires (1992), since the world is not static, there apparently exists an incompatibility between the Hartle-Hawking proposal and the Bohmian model. What Squires ignores is the fact that Hartle-Hawking interpret the wave function only in the classical region of minisuperspace, where it looks like the sum of two WKB solutions. Hartle-Hawking interpret this as two non-interfering descriptions of our universe. Though we wonder whether breaking the superposition is justified, if it is (as has been claimed), their proposal is compatible with the present approach: their wave function corresponds to two non-interfering sets of trajectories. If it is unjustified, then the Hawking-Hartle conjecture is physically uninteresting, and our proposal's apparent incompatibility with it does not bother us. The more general issue of real wave functions in the causal theory is rather complicated, and shall not be discussed here.

The present model should be distinguished from another recent suggestion in the Bohmian spirit, namely, Pitowsky (1991). Pitowsky essentially adds a quantum correction term to the classical gravitational field equations, as Bohm adds a quantum potential to the Hamilton-Jacobi equation. It is in that sense that the proposal is Bohmian. Unlike in QG (and thus the present proposal), Pitowsky does not quantize the metric field; that is, the wave function is still a function of x, not of h(x). The geometry is affected only through the new quantum input into the momentum-energy tensor. Consequently, Pitowsky's project is more in line with the semiclassical approach to QG, in which the matter but not the metric fields are quantized and described by Schrodinger's equation. As such, we expect it to suffer from similar difficulties, e.g., the well-known problem posed by superpositions (see Kuchar 1992). Because he has not addressed the issue of how time and dynamics arise, it is hard to say exactly how the problem of time will manifest itself for his proposal.[4] Anyway, since there are reasons for believing that the semiclassical approach is unsatisfactory, Pitowsky's approach, though interesting, is not especially promising.

9. Time in a Bohmian Universe

The time parameter labeling the trajectories in Bohmian cosmology is a theoretical posit. Like Newton's absolute time, the time in Bohmian cosmology is most naturally viewed as an unobservable, physical time, arising from the basic laws. We stress that this claim is only speculation. Clearly, a successful integration of quantum mechanics and general relativity, for instance, would demand reevaluation. Nevertheless, the picture described is the most straightforward one.

Consider the status of time in Newtonian mechanics. Newton believed time was a real relation, not be confused with its sensible measure. He also presumed it to be unique and empirically determined. The reason is that from an independent knowledge of the true forces of a moving body, a single time measure consistent with his mechanics emerges. Change this time, and what was once (for instance) a freely moving body with no forces acting on it becomes an accelerating/decelerating body apparently suffering the imposition of forces. Since Newtonian theory already tells us what the forces are, e.g., in simple physical situations, the true measure of time can be detected. Since it follows from the laws of physics, there is a clear sense in which the unique time is empirically determined. While this reasoning may be epistemically circular (for the selection of forces may originate from the selection of time, or vice versa), it is nonvicious, for it is probably endemic to the practice of theorizing.

The situation seems similar in Bohmian cosmology. Here time also looks to be a theoretical posit. The t in equations (7) cannot be directly measured; yet from a knowledge of the true 'forces' acting on the cosmological variables, t is uniquely determined (that is, if the laws of Bohmian cosmology are correct). If this is correct, Bohmian cosmology is not generally covariant. The laws define a preferred time. We would like to make two remarks on this consequence. First, though a preferred time may be upsetting in QG, it is not from the perspective of quantum cosmology. In cosmology, the goal is to watch various quantities evolve with respect to cosmic time. Bohmian cosmology allows for precisely this. If it does not apply to the infinite number of time parametrizations (most of which are pathological) compatible with general relativity, is much of value lost to cosmology?

Second, we should squarely face the fact that this consequence may be inevitable anyway. The so-called problem of functional evolution seriously threatens the covariance of general relativity (Kuchar 1988, 227). The problem stems from the possibility that the commutators between the constraint operators may not vanish:

$$[H(x), H'(x)] \neq 0.$$

If that obtains, a state's evolution from an initial hypersurface to a final one might depend on the foliation connecting the two hypersurfaces. One state on the initial hypersurface might be developed into two nonequivalent states on the later (earlier) hypersurface. This problem is often overlooked because it obtains only in models that are infinitely dimensional. Nevertheless, since the difficulty is equivalent to the notorious factor-ordering problem, it should be taken quite seriously. Fixing the spacetime's foliation solves the problem, of course, but only at the expense of covariance. Critics of the non-covariance of Bohmian cosmology should thus bear in mind the possibility that their favorite theory may someday share the same fate.

Finally, a crucial question (perhaps *the* crucial question) confronting all quantum cosmological schemes remains to be addressed. The question is: is the time variable posited in quantum cosmology identical to the one referred to in the rest of physics, e.g., in Schrodinger's equation? Assuming there is at most one physical time, the question asks whether quantum cosmology and the rest of physics are mutually consistent.

So far as we can detect, the answer for Bohmian cosmology is uncertain, but quite promising. First, the time function found within the region of configuration space where the WKB approximation is valid is the same as that in the time-dependent Schrodinger equation (see e.g. Banks 1985). Due to the formal similarity between WKB and the present endeavor, then, the time function posited in section 7 is the same as the one found in the Schrodinger equation. Whether it is the same with a different wave function is unknown. It has been suggested that the concept of time is semiclassical, and breaks down outside the WKB region. If this is the case, then the Bohmian time is the 'right' time; if it is not so, and there are meaningful time functions outside the WKB region, then it is an open question whether they are all equivalent.

Second, the cosmic time posited by Bohmian cosmology is at least the right kind of entity to be identified with the time of dynamics. This feature is one happy consequence of Bohmian cosmology's implicit rejection of general relativity. Time in Bohmian cosmology seems to refer to real temporal relations, of the sort intimately associated with mechanics. It is not the arbitrary parameter found in general relativity. Prima facie, Bohmian time is more plausibly identified with the time of dynamics than with that found in many other proposals.

Notes

[1]Consider a universe at t=0 which is in a superposition between two cosmological properties, $\Psi = |a1\rangle + |a2\rangle$. It is now at t = present found to be in state (say) $|a2\rangle$.

Then we know with certainty that any measurement of Ψ between t=0 and t=present would have yielded |a2>. This fact goes unexplained in the orthodox interpretation. But in Bohm's theory, the reason is clear: the universe exists independently of the wave function, and was in state |a2> the entire duration. See Aharonov and Albert (1987).

[2]Gell-Mann and Hartle (1989) obtain a classical limit with their coarse-grained projection operators, though there remains the question of whether these can be defined without violating quantum mechanics (see Bohm and Hiley 1993, ch.14).

[3]If the space allows a timelike Killing vector field and a non-negative potential, the inner product will be positive. Unfortunately, neither condition is likely to be satisfied in QG. Theorems due to Kuchar make it plausible that QG does not admit the relevant isometries for a timelike Killing vector field. Additionally, the QG potential can be both negative and positive (and of course, the solutions cannot be restricted to the positive energy ones, for these correspond to the physically significant contraction of the 3-volume).

[4]If he introduces time like others in the semiclassical approach, he must isolate some classical degrees of freedom to play the role of time. Such a split is notoriously difficult (see Unruh and Wald 1989). If he tries to manage without time, say with a path integral calculating the transition amplitudes between states (as he indicates he might, p.349), then the integral typically depends on choice of foliation, and additionally leads to violations of the Hamiltonian constraints; see Kuchar (1992) for details.

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